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## DISTRIBUTION AND MOMENTS OF RADIAL ERROR

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## TABLE OF CONTENTS

	Page
SUMMARY .....	1
I. INTRODUCTION.....	1
II. FIRST SPECIAL CASE .....	7
III. SECOND SPECIAL CASE .....	9
IV. A NOTE ON THE N-DIMENSIONAL CASE .....	11
APPENDIX — MOMENTS, DENSITY, AND DISTRIBUTION FUNCTION GRAPHS AND TABLES .....	15
REFERENCES .....	50
BIBLIOGRAPHY.....	51

## LIST OF ILLUSTRATIONS

Figure	Title	Page
A-1.	Central moments versus $\beta$ when $\bar{X} = \bar{Y} = 0$ , $\beta = \sigma_X/\sigma_Y$ , $\mu$ = mean, $\mu_2$ = variance .....	16
A-2.	Central moments versus $\lambda$ when $\sigma_X = \sigma_Y = \sigma$ , $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ , $\mu$ = mean, $\mu_2$ = variance, $\sqrt{\mu_2}$ = standard deviation .....	17
A-3.	Distribution of standardized variable; $t = r/\sigma$ , when $\sigma_X =$ $\sigma_Y = \sigma$ , and $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ .....	18
A-4.	$g(t) = \beta t e^{-\frac{t^2}{4}(\beta^2+1)} I_0\left[\frac{t^2}{4}(\beta^2-1)\right]$ , for $\beta = 1, 1.5, 2, 3$ , and 4 where $\beta = \sigma_X/\sigma_Y$ .....	19
A-5.	$g(t) = t e^{-(\lambda^2+t^2)} I_0(\lambda t)$ , for $\lambda = 0, 1, 2$ , and 3 where $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ .....	20

## LIST OF TABLES

Table	Title	Page
A-1.	Noncentral Moments When $\bar{X} = \bar{Y} = 0$ , $\beta = \frac{\sigma_X}{\sigma_Y}$ .....	21
A-2.	Central Moments When $\bar{X} = \bar{Y} = 0$ , $\beta = \frac{\sigma_X}{\sigma_Y}$ .....	23
A-3.	Probability Distribution Function When $\bar{X} = \bar{Y} = 0$ , $\beta = \frac{\sigma_X}{\sigma_Y}$ ; $F(s) = \int_0^s \beta t e^{-\frac{1}{4}(\beta^2+1)t^2} I_0\left[\frac{(\beta^2-1)t^2}{4}\right] dt$ .....	25
A-4.	Noncentral Moments When $\sigma_X = \sigma_Y = \sigma$ , $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ .....	30
A-5.	Central Moments When $\sigma_X = \sigma_Y = \sigma$ , $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ .....	34
A-6.	Probability Distribution When $\sigma_X = \sigma_Y = \sigma$ , $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ ; $F(s) = \int_0^s t e^{-\frac{1}{2}(t^2 + \lambda^2)} I_0(\lambda t) dt$ .....	38

## DISTRIBUTION AND MOMENTS OF RADIAL ERROR

## SUMMARY

The distribution and moments of  $r = (X^2 + Y^2)^{1/2}$ , where  $X$  and  $Y$  are normally distributed random variables, are considered in this report. This distribution is the so-called generalized Rayleigh distribution which has many applications in the study of wind shear, random noise, and radar.

Section I, Introduction, is a derivation of the most general formula in the two-dimensional case, following the work of Weil [1], Yadavalli [2], Miller et al.[3], and Smith [4]. Two special cases are then considered, for which tabulation of the moments and probability distribution functions is included as an appendix. Finally, following Miller [5], the multidimensional case is considered in Section IV, where a somewhat specialized formula is derived.

## I. INTRODUCTION

Let  $X_1$  and  $Y_1$  be normally distributed random variables. Since independent (uncorrelated) variables simplify computation considerably, the axes are rotated by the appropriate angle  $\phi$  to obtain zero correlation between  $X$  and  $Y$ . This transformation is given by

$$\begin{aligned} X &= X_1 \cos \phi + Y_1 \sin \phi \\ Y &= -X_1 \sin \phi + Y_1 \cos \phi \end{aligned} \quad , \quad (1)$$

where  $\tan \phi = 2\rho\sigma_1\sigma_2/(\sigma_1^2 + \sigma_2^2)$  and  $\rho$  is the correlation coefficient between  $X_1$  and  $Y_1$ ;  $\sigma_1$  and  $\sigma_2$  are the respective standard deviations of  $X_1$  and  $Y_1$ .

Now,  $X$  and  $Y$  have the bivariate normal distribution given by

$$f(X, Y) = \frac{1}{2\pi\sigma_X\sigma_Y} e^{-\frac{1}{2} \left[ \left( \frac{X - \bar{X}}{\sigma_X} \right)^2 + \left( \frac{Y - \bar{Y}}{\sigma_Y} \right)^2 \right]}, \quad (2)$$

where  $\bar{X}$ ,  $\bar{Y}$  and  $\sigma_X^2, \sigma_Y^2$  are the means and variances of  $X$  and  $Y$ .

Now, change to polar coordinates and integrate over 0 to  $2\pi$  to get the marginal distribution of  $r = (X^2 + Y^2)^{1/2}$ . Let  $X = r \cos \theta$  and  $Y = r \sin \theta$  in equation (2) and use the identities  $\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$  and  $\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$ . After simplifying, we have

$$g(r) = \frac{\alpha_0 r e^{-\alpha_1 r^2}}{2\pi} \int_0^{2\pi} e^{r^2 \cos 2\theta + ar \cos \theta + br \sin \theta} d\theta, \quad (3)$$

where

$$\alpha_0 = \frac{e^{-\frac{1}{2} \left( \frac{\bar{X}^2}{\sigma_X^2} + \frac{\bar{Y}^2}{\sigma_Y^2} \right)}}{\sigma_X \sigma_Y},$$

$$\alpha_1 = \frac{\sigma_X^2 + \sigma_Y^2}{4\sigma_X^2 \sigma_Y^2},$$

$$\alpha_2 = \frac{\sigma_X^2 - \sigma_Y^2}{4\sigma_X^2 \sigma_Y^2},$$

$$a = \frac{\bar{X}}{\sigma_X^2},$$

$$b = \frac{\bar{Y}}{\sigma_Y^2},$$

and  $r$  is the Jacobian of the transformation.

From the well-known generating function

$$e^{Z(t - 1/t)} = \sum_{n=-\infty}^{\infty} J_n(Z) t^n,$$

we obtain the identities

$$e^{Z \sin \theta} = \sum_{n=-\infty}^{\infty} I_n(Z) e^{ni\left(\theta + \frac{3\pi}{2}\right)} \quad (4)$$

and

$$e^{Z \cos \theta} = \sum_{n=-\infty}^{\infty} I_n(Z) e^{ni\theta} \quad . \quad (5)$$

Substitution of equations (4) and (5) into equation (3) yields

$$g(r) = \frac{\alpha_0 r e^{-\alpha_1 r^2}}{2\pi} \int_0^{2\pi} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I_k(r^2 \alpha_2) I_m(ar) I_n(br) e^{i\theta(2k+m+n) + \frac{3ni\pi}{2}} d\theta \quad . \quad (6)$$

Since each of the summations in the integrand converges uniformly, the order of integration and summation may be interchanged. Also, since  $g(r)$  is purely real, the imaginary component in equation (6) must vanish. Equation (6) is real when  $2k+m+n=0$  and  $n=2\ell$  for integer  $\ell$ . Hence,  $-m=2k+2\ell$  and, noting that  $e^{3\ell i\pi} = (-1)^\ell$ , equation (6) becomes

$$g(r) = \alpha_0 r e^{-\alpha_1 r^2} \sum_{k=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} (-1)^\ell I_k(r^2 \alpha_2) I_{-2k-2\ell}(ar) I_{2\ell}(br) . \quad (7)$$

From Watson [6] we find the identity (which is a form of Graf's formula)

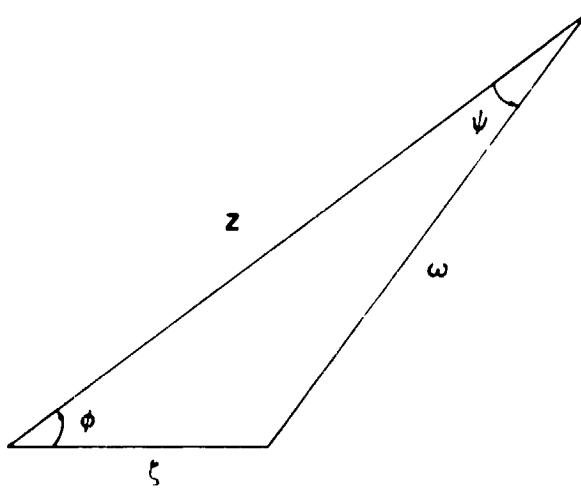
$$I_\nu(\omega) \cos \nu\psi = \sum_{m=-\infty}^{\infty} (-1)^m I_{\nu+m}(z) I_m(z) \cos m\phi ,$$

where

$$z - \xi \cos \phi = \omega \cos \psi$$

and

$$\xi \sin \phi = \omega \sin \psi$$



Letting  $\phi = \pi/2$ ,  $Z = ar$ , and  $\xi = br$  results in  $\omega = r(a^2 + b^2)^{1/2}$  and  $\tan \psi = b/a$ , and using the fact that  $I_{-n}(Z) = I_n(Z)$ , equation (7) may be written as

$$g(r) = \alpha_0 r e^{-\alpha_1 r^2} \left[ I_0(\alpha_2 r^2) I_0(\alpha_3 r) + 2 \sum_{k=1}^{\infty} I_k(\alpha_2 r^2) I_{2k}(\alpha_3 r) \cos 2k\psi \right], \quad (8)$$

where

$$\alpha_0 = \frac{e}{\sigma_X \sigma_Y} \left( -\frac{1}{2} \left( \frac{\bar{X}^2}{\sigma_X^2} + \frac{\bar{Y}^2}{\sigma_Y^2} \right) \right),$$

$$\alpha_1 = \frac{\sigma_X^2 + \sigma_Y^2}{4\sigma_X^2 \sigma_Y^2},$$

$$\alpha_2 = \frac{\sigma_X^2 - \sigma_Y^2}{4\sigma_X^2 \sigma_Y^2},$$

$$\alpha_3 = \left[ \left( \frac{\bar{X}}{\sigma_X} \right)^2 + \left( \frac{\bar{Y}}{\sigma_Y} \right)^2 \right]^{1/2},$$

$$\tan \psi = \frac{\bar{Y} c_X^2}{\bar{X} \sigma_Y^2},$$

and  $I_k(Z)$  denotes the modified Bessel function of the first kind of order  $k$ .

We have again listed  $\alpha_0$ ,  $\alpha_1$ , and  $\alpha_2$  for reference.

Now equation (8) holds for our rotated (independent) variables; to obtain the analogous expression for  $r_1 = (X_1^2 + Y_1^2)^{1/2}$  where  $X_1$  and  $Y_1$  may be correlated, apply the inverse transformation of equation (1) and, after some algebraic simplification,

$$g(r_1) = \alpha_0 r e^{-a_1 r_1^2} \left[ I_0(a_2 r_1^2) I_0(\alpha_3 r_1) + 2 \sum_{k=1}^{\infty} I_k(a_2 r_1^2) I_{2k}(\alpha_3 r_1) \cos 2k\psi \right]; \quad (9)$$

$$a_1 = \frac{\sigma_1^2 + \sigma_2^2}{4(1-\rho^2)\sigma_1^2 \sigma_2^2},$$

$$a_2 = \frac{\sigma_1^2 - \sigma_2^2 + 4\rho^2 \sigma_1^2 \sigma_2^2}{4(1-\rho^2)\sigma_1^2 \sigma_2^2},$$

$$\sigma_X \sigma_Y = \sigma_1 \sigma_2 (1-\rho^2)^{1/2},$$

$$\sigma_X^2 + \sigma_Y^2 = \sigma_1^2 + \sigma_2^2,$$

and

$$\sigma_{(1,2)}^2 = \frac{1}{2} \{ \sigma_X^2 + \sigma_Y^2 \pm [(\sigma_X^2 + \sigma_Y^2)^2 - 4\sigma_X^2 \sigma_Y^2 (1-\rho^2)]^{1/2} \},$$

where  $\sigma_1$  takes the positive root,  $\sigma_2$  the negative root, and  $\alpha_0$ ,  $\alpha_3$ , and  $\tan \psi$  are as explained previously.

Because of the abundance of parameters in these most general cases, tabulation of the probability distribution functions and the moments is not practical. Instead, two important special cases were selected in which these statistics could be conveniently tabulated.

## II. FIRST SPECIAL CASE

Let  $X$  and  $Y$  be independent Gaussian random variables having zero means,  $\bar{X} = \bar{Y} = 0$ . Then from equation (8) we have

$$g(r) = \frac{r}{\sigma_X \sigma_Y} e^{-\left(\frac{\sigma_X^2 + \sigma_Y^2}{4\sigma_X^2 \sigma_Y^2}\right)r^2} I_0 \left[ \left( \frac{\sigma_X^2 - \sigma_Y^2}{4\sigma_X^2 \sigma_Y^2} \right) r^2 \right] \quad (10)$$

and  $\mu'_k$ , the  $k$ th noncentral moment, can be written as

$$\mu'_k = \int_0^\infty \frac{r^{k+1}}{\sigma_X \sigma_Y} e^{-\frac{1}{4} \left( \frac{\sigma_X^2}{\sigma_Y^2} + 1 \right) \frac{r^2}{\sigma_X^2}} I_0 \left[ \left( \frac{\sigma_X^2}{\sigma_Y^2} - 1 \right) \frac{r^2}{\sigma_X^2} \right] dr . \quad (11)$$

Letting  $\beta = \sigma_X/\sigma_Y$  and making the change of variable  $t = r/\sigma_X$ , equation (11) becomes

$$\mu'_k = \sigma_X^k \beta \int_0^\infty t^{k+1} e^{-\frac{(\beta^2+1)}{4} t^2} I_0 \left[ \frac{(\beta^2-1)t^2}{4} \right] dt , \quad (12)$$

which from Gradshteyn and Ryzhik [7] or Miller [5] is expressed in closed form as

$$\mu'_k = \sigma_X^k \beta 2^{-k} (\beta^2+1)^{-\frac{k+2}{2}} \Gamma\left(\frac{k+2}{2}\right) {}_2F_1 \left[ \frac{k+2}{4}, \frac{k+4}{4}; 1; \left( \frac{\beta^2-1}{\beta^2+1} \right)^2 \right] \quad 0 < \beta < \infty , \quad (13)$$

where  $\Gamma(Z)$  is the well-known gamma function and  ${}_2F_1(a, b; c; Z)$  denotes the hypergeometric function. The central moments can be found by using the recurrence relation

$$\mu_k = \mu'_k - k\mu'_{k-1}\mu + \dots + (-1)^i \binom{k}{i} \mu'_{k-i}\mu^i + \dots + (-1)^{k-1} (k-1)\mu^k , \quad (14)$$

where  $\mu_k'$  is the  $k$ th central moment and  $\mu = \mu'_1$  is the mean. Explicitly,

$$\mu = \mu'_1$$

$$\mu_2 = \mu'_2 - \mu^2$$

(15)

$$\mu_3 = \mu'_3 - 3\mu'_2\mu + 2\mu^3$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu + 6\mu'_2\mu^2 - 3\mu^4 .$$

The values of  $\mu$ ,  $\mu_2$ ,  $\mu_3$ ,  $\mu_4$ ,  $\mu'_2$ ,  $\mu'_3$ ,  $\mu'_4$ ,  $\sqrt{\mu_2}$ , and  $\mu/\sqrt{\mu_2}$  are tabulated in the appendix (Tables A-1 and A-2) for unit variance in  $X$ , ( $\sigma_X = 1$ ). Thus, the tabulated values of  $\mu_k$ ,  $\mu'_k$  for  $k = 1, 2, 3, 4$  must be multiplied by  $\sigma_X^k$ , the standard deviation  $\sqrt{\mu_2}$  must be multiplied by  $\sigma_X$ , and the coefficient of variation  $\mu/\sqrt{\mu_2}$  is a unitless quantity which is correct as tabulated.

To determine the probability distribution function of  $r$ , make the change of variable  $t = r/\sigma_X$  in equation (10) and, as before, let  $\beta = \sigma_X/\sigma_Y$ . Then,  $\left| \frac{dr}{dt} \right| = \sigma_X$  and

$$g(t) = \beta t e^{-\frac{1}{4}(\beta^2+1)t^2} I_0\left[\frac{(\beta^2-1)t^2}{4}\right] \quad (16)$$

and the probability distribution function of  $t$  is

$$F(s) = \beta \int_0^s t e^{-\frac{1}{4}(\beta^2+1)t^2} I_0\left[\frac{(\beta^2-1)t^2}{4}\right] dt . \quad (17)$$

Values of  $F(s)$  for different values of  $\beta$  and  $s$  have been tabulated in Table A-3. To utilize the table, make use of the conversion formula

$$\Pr(r \leq s) = \Pr\left(t \leq \frac{s}{\sigma_X}\right) = F\left(\frac{s}{\sigma_X}\right) . \quad (18)$$

Note that in all the tables mentioned, only values of  $\beta \geq 1$  have been tabulated since it is always possible to assume  $\sigma_X \geq \sigma_Y$ . Another special case will now be considered for which tabulation of the various statistics was convenient and valuable.

### III. SECOND SPECIAL CASE

Let  $X$  and  $Y$  be independent random variables having a normal distribution with equal variances,  $\sigma_X = \sigma_Y = \sigma$ . Then, from equation (8)

$$g(r) = \frac{r}{\sigma^2} e^{-\frac{(\bar{X}^2 + \bar{Y}^2)}{2\sigma^2}} e^{-\frac{r^2}{2\sigma^2}} I_0\left[\frac{(\bar{X}^2 + \bar{Y}^2)^{1/2}}{\sigma^2} r\right] . \quad (19)$$

The  $k$ th noncentral moment of  $r$  may then be written as

$$\mu'_k = \frac{e^{-\frac{(\bar{X}^2 + \bar{Y}^2)}{2\sigma^2}}}{\sigma^2} \int_0^\infty r^{k+1} e^{-\frac{r^2}{2\sigma^2}} I_0\left[\frac{(\bar{X}^2 + \bar{Y}^2)^{1/2}}{\sigma^2} r\right] dr , \quad (20)$$

which from Gradshteyn and Ryzhik [7] is, after some simplification,

$$\mu'_k = e^{-\frac{(\bar{X}^2 + \bar{Y}^2)}{2\sigma^2}} (2\sigma^2)^{k/2} \Gamma\left(\frac{k+2}{2}\right) {}_1F_1\left(\frac{k+2}{2}; 1; \frac{\bar{X}^2 + \bar{Y}^2}{2\sigma^2}\right), \quad (21)$$

where  ${}_1F_1(a;b;Z)$  denotes the confluent hypergeometric function (Kummer's function). Let  $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ ; then equation (21) becomes

$$\mu'_k = e^{-\lambda^2/2} (2\sigma^2)^{k/2} \Gamma\left(\frac{k+2}{2}\right) {}_1F_1\left(\frac{k+2}{2}; 1; \frac{\lambda^2}{2}\right). \quad (22)$$

Again the central moments can be found using equations (14) and (15). It might be noted that the second noncentral moment simplifies considerably to  $\mu'_2 = \sigma^2(2+\lambda^2)$ .

Values of  $\mu'_k$ ,  $\mu_k$  for  $k = 1, 2, 3, 4, \sqrt{\mu_2}$ , and  $\mu/\sqrt{\mu_2}$  have been tabulated in Tables A-4 and A-5 for  $\sigma = 1$ . Thus, the tabulated values of  $\mu'_k$  and  $\mu_k$  must be multiplied by  $\sigma^k$ ,  $\sqrt{\mu_2}$  by  $\sigma$ , and  $\mu/\sqrt{\mu_2}$  is correct as tabulated.

To find the probability distribution function, let  $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$  in equation (19) and make a change to the standardized variable  $t = r/\sigma$ . Then,  $\left|\frac{dr}{dt}\right| = \sigma$  and

$$F(s) = e^{-\lambda^2/2} \int_0^s t e^{-t^2/2} I_0(\lambda t) dt. \quad (23)$$

Values of  $F(s)$  have been tabulated in Table A-6 for various values of  $s$  and  $\lambda$ . Since this is the probability distribution of  $t$ , when using the tables utilize the formula

$$\Pr(r \leq s) = \Pr\left(t \leq \frac{s}{\sigma}\right) = F\left(\frac{s}{\sigma}\right). \quad (24)$$

When  $\lambda = 0$ , the distribution of  $r$  is that of the "classical" Rayleigh

$$g(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

and has mean  $\sigma\sqrt{\pi/2}$  and variance  $2\sigma^2 - \frac{\sigma^2\pi}{2}$ . This coincides exactly with the case where  $\bar{X} = \bar{Y} = 0$  and  $\beta = 1$ . The graphs of the density functions have been supplied in the appendix for both special cases for various values of the parameters  $\beta$  and  $\lambda$ , assuming unit variance in  $X$  and  $Y$ .

We now discuss a generalization of this special case to  $n$ -dimensions, essentially the only case where such a generalization has been completely successful.

#### IV. A NOTE ON THE N-DIMENSIONAL CASE

Let  $X_n = \{x_1, x_2, \dots, x_n\}$  be a random column vector, where the  $x_i$  are pairwise independent Gaussian random variates, with mean vector  $A_n = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n\}$  and positive definite diagonal covariance matrix  $\sigma^2 E_n$ , when  $E_n$  is the  $n \times n$  identity matrix. We wish to determine the density function of  $r = (x_1^2 + x_2^2 + \dots + x_n^2)^{1/2}$ . The density function of  $X_n$  is given by

$$f(X_n) = \frac{1}{(2\pi)^{n/2} \sigma^n} e^{-\frac{1}{2\sigma^2} (X_n - A_n)'(X_n - A_n)}, \quad (25)$$

where the prime denotes matrix transposition. Notice that

$$(X_n - A_n)'(X_n - A_n) = (r^2 + a^2) - 2X_n' A_n,$$

where

$$a^2 = \bar{x}_1^2 + \bar{x}_2^2 + \dots + \bar{x}_n^2 ;$$

thus,

$$f(\bar{X}_n) = \frac{e^{-\frac{(r^2 + a^2)}{2\sigma^2}}}{(2\pi)^{n/2} \sigma^n} e^{\frac{1}{\sigma^2} \bar{X}_n' A_n} = \gamma e^{\frac{1}{\sigma^2} \bar{X}_n' A_n} . \quad (26)$$

Now, introduce a complete orthonormal set of vectors  $\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n$  so that  $\vec{e}_1$  is in the direction of  $A_n$ . If  $\phi_1$  is the angle between  $A_n$  and  $X_n$  and  $\| \cdot \|$  denotes norm, we have the inner product

$$X_n' A_n = \|X_n\| \|A_n\| \cos \phi_1 = r a \cos \phi_1 .$$

Now, make the change to polar coordinates and integrate over the range of the  $n-1$  angles  $\phi_1, \phi_2, \dots, \phi_{n-1}$ . The transformation is given by

$$\left. \begin{aligned} x_i &= r \cos \phi_i \prod_{j=1}^{i-1} \sin \phi_j \quad \text{for } 1 \leq i \leq n-1 \\ x_n &= r \prod_{j=1}^{n-1} \sin \phi_j \end{aligned} \right\} \begin{aligned} 0 &\leq \phi_i \leq \pi \quad \text{for } 0 \leq i \leq n-2 \\ 0 &\leq \phi_{n-1} \leq 2\pi \end{aligned}$$

and the Jacobian

$$J_n = r^{n-1} \prod_{k=1}^{n-2} \sin^{n-1-k} \phi_k .$$

The density function of  $r$  is thus given by

$$g(r) = \gamma r^{n-1} \int_0^\pi e^{-\frac{ra \cos \phi_1}{\sigma^2}} \sin^{n-2} \phi_1 d\phi_1 \prod_{j=2}^{n-2} \int_0^\pi \sin^{n-1-j} \phi_j d\phi_j \int_0^{2\pi} d\phi_{n-1} . \quad (27)$$

From Watson [6]

$$I_\nu(Z) = \frac{\left(\frac{Z}{2}\right)^\nu}{\Gamma\left(\nu + \frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right)} \int_0^\pi e^{\pm Z \cos \theta} \sin^{2\nu} \theta d\theta$$

and from Magnus, Oberhettinger, and Soni [8],

$$\int_0^\pi \sin^{n-1-k} \phi_k d\phi_k = B\left(\frac{n-k}{2}, \frac{1}{2}\right) = \frac{\Gamma\left(\frac{n-k}{2}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n-k+1}{2}\right)} ,$$

where  $B(a, b)$  is the Beta function.

Applying these identities to  $g(r)$ ,

$$g(r) = \frac{2\pi e^{-\frac{(r^2+a^2)}{2\sigma^2}} r^{n-1} \Gamma\left(\frac{n-1}{2}\right) \Gamma\left(\frac{1}{2}\right) I_{\frac{n-2}{2}}\left(\frac{ra}{2\sigma^2}\right)}{(2\pi)^{n/2} \sigma^n \left(\frac{ra}{2\sigma^2}\right)^{(n+2)/2} \prod_{j=2}^{n-2} \frac{\Gamma\left(\frac{n-j}{2}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n-k+1}{2}\right)}} , \quad (28)$$

but

$$\prod_{j=2}^{n-2} \frac{\Gamma\left(\frac{n-j}{2}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n-k+1}{2}\right)} = \frac{\Gamma^{n-3}\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)},$$

which simplifies equation (28) to

$$g(r) = \frac{a}{\sigma^2} \left(\frac{r}{a}\right)^{n/2} e^{-\frac{(r^2 + a^2)}{2\sigma^2}} I_{\frac{n-2}{2}}\left(\frac{ra}{2\sigma^2}\right). \quad (29)$$

To find  $\mu'_k$ , the  $k$ th noncentral moment, we must evaluate  $\int_0^\infty r^k g(r) dr$ , which from the same identity that was used in Section III gives

$$\mu'_k = \frac{(2\sigma^2)^{k/2} e^{-a^2/2\sigma^2} \Gamma\left(\frac{k+n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} {}_1F_1\left(\frac{n+k}{2}; \frac{n}{2}; \frac{a^2}{2\sigma^2}\right); \quad (30)$$

of course the central moments are found in standard fashion using equation (14).

The analogous expression to equation (29) when  $X_n$  has an arbitrary positive definite diagonal covariance matrix could not be found in closed form; however, the interested reader is referred to the paper of Blumenson and Miller [9] where a symbolic expression is obtained.

**APPENDIX**  
**MOMENTS, DENSITY, AND DISTRIBUTION  
FUNCTION GRAPHS AND TABLES**

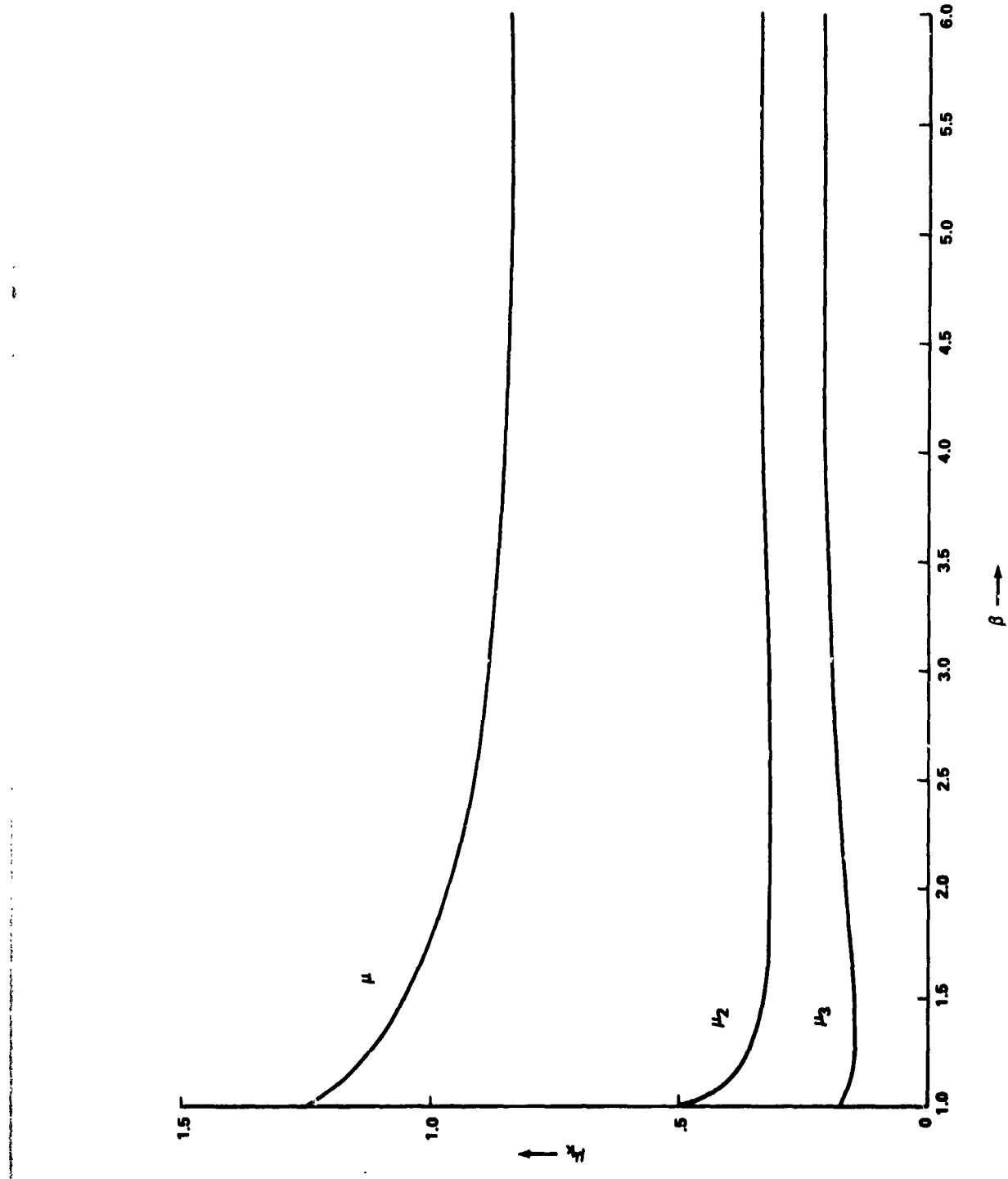


Figure A-1. Central moments versus  $\beta$  when  $\bar{X} = \bar{Y} = 0$ ,  $\beta = \sigma_X/\sigma_Y$ ,  $\mu$  = mean,  $\mu_2$  = variance.

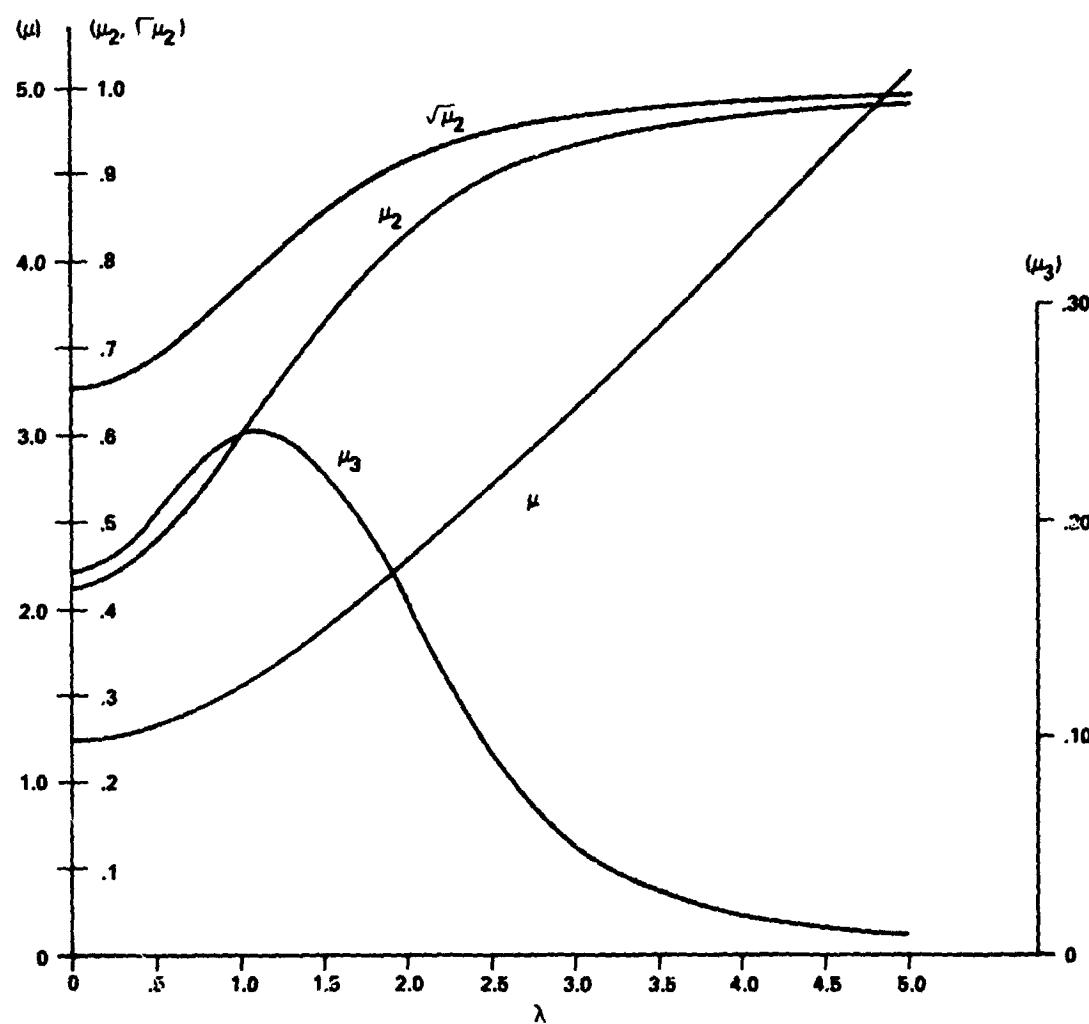


Figure A-2. Central moments versus  $\lambda$  when  $\sigma_X = \sigma_Y = \sigma$ ,  $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$   
 $\mu$  = mean,  $\mu_2$  = variance,  $\sqrt{\mu_2}$  = standard deviation.

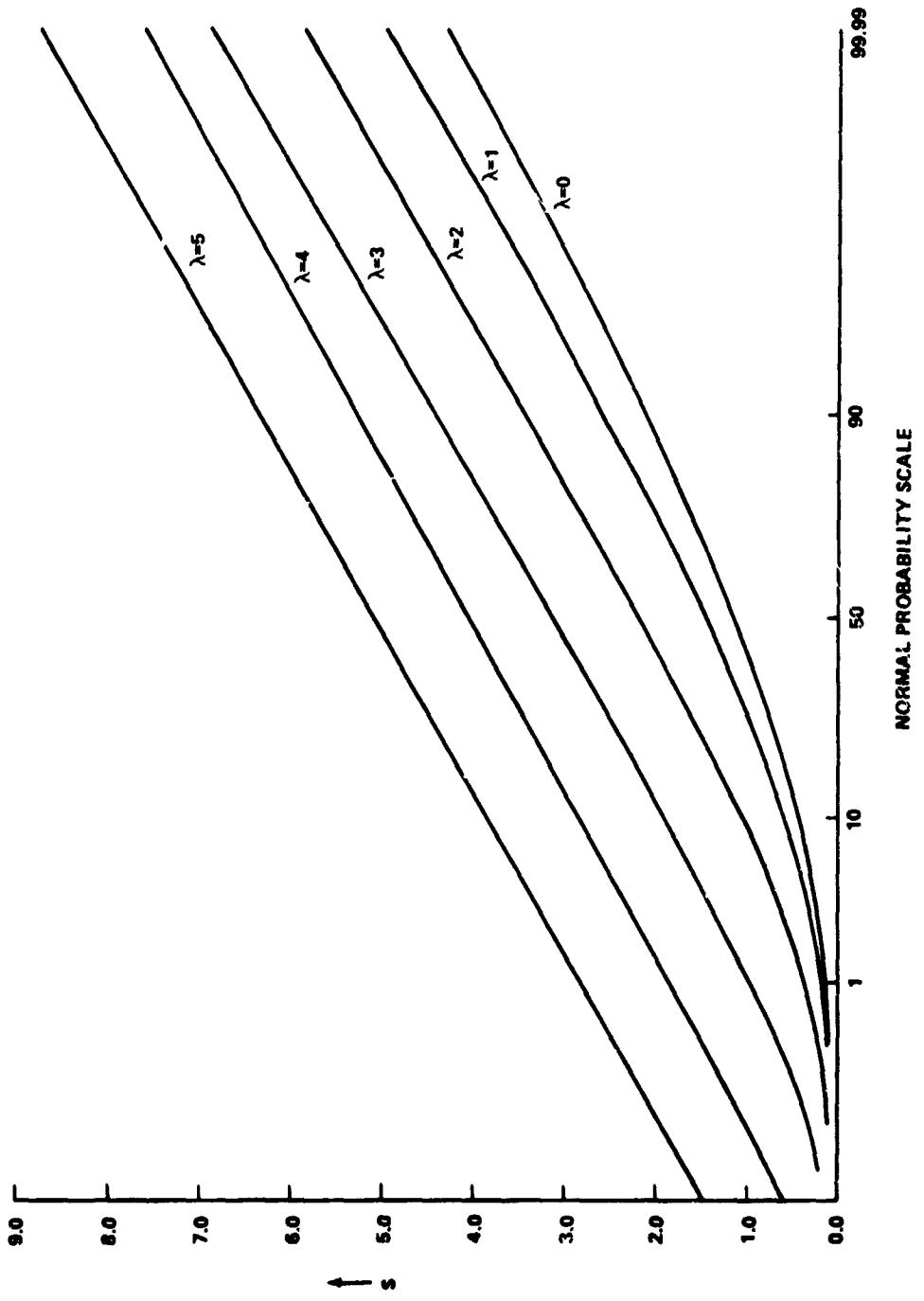


Figure A-3. Distribution of standardized variable;  $t = r/\sigma$ , when  $\sigma_X = \sigma_Y = \sigma$ , and  $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ .

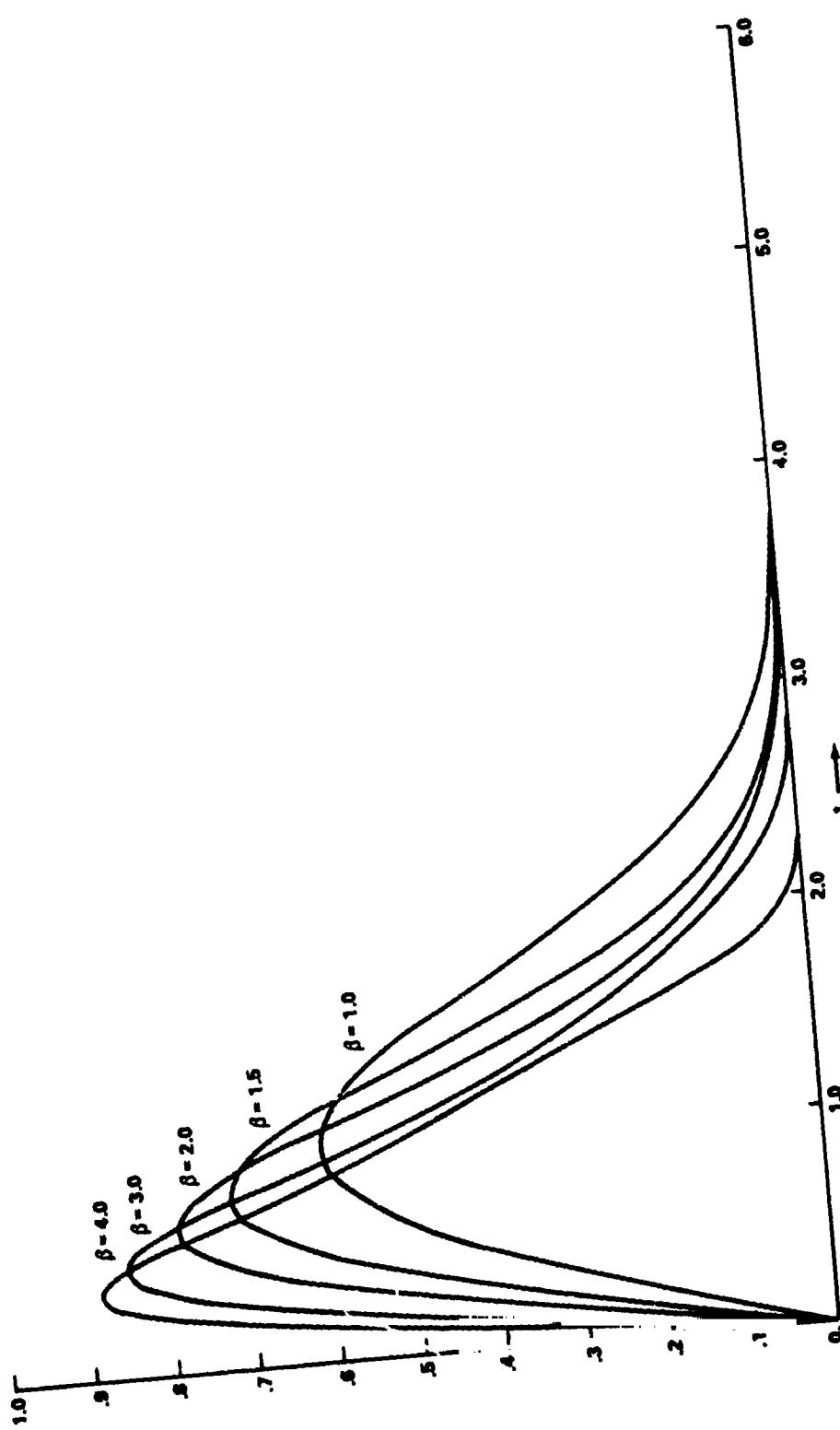


Figure A-4.  $g(t) = \beta t e^{-\frac{t^2}{4}} I_0 \left[ \frac{t^2}{4} (\beta^2 - 1) \right]$ , for  $\beta = 1, 1.5, 2, 3$ , and  $4$  where  $\beta = \sigma_X / \sigma_Y$ .

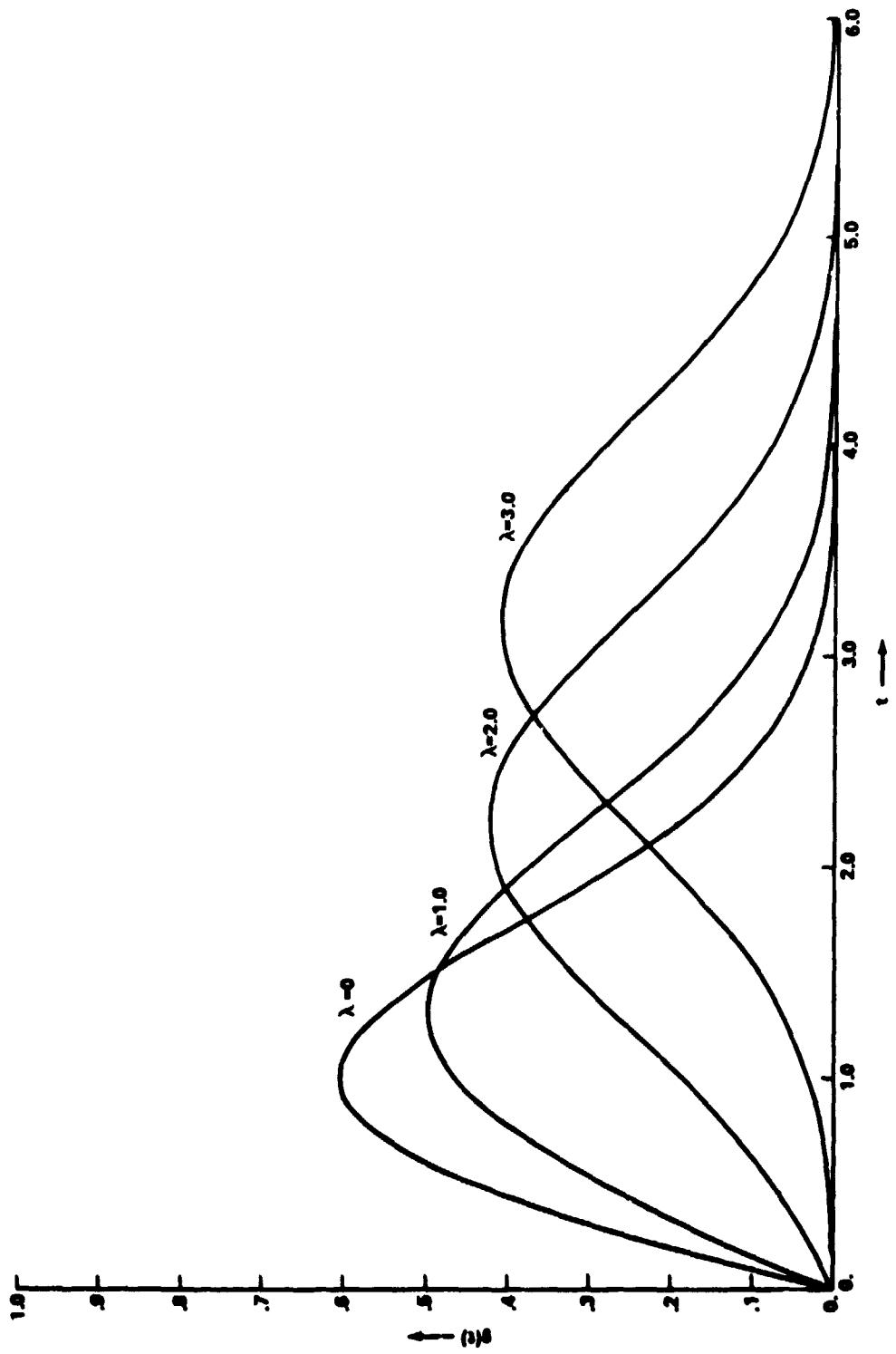


Figure A-5.  $g(t) = t e^{-(\lambda^2 + t^2)} I_0(\lambda t)$ , for  $\lambda = 0, 1, 2$ , and  $3$  where  $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$ .

TABLE A-1. NONCENTRAL MOMENTS WHEN  $\bar{X} = \bar{Y} = 0$ ,  $\beta = \frac{\sigma_X}{\sigma_Y}$

MEAN X=MEAN Y=0, SIGMA X/SIGMA Y=BETA

$\beta$	1st Noncentral Moment	2nd Noncentral Moment	3rd Noncentral Moment	4th Noncentral Moment
1.00	1.25331	2.00000	3.75994	8.13000
1.05	1.22366	1.93733	3.50240	7.24217
1.10	1.19702	1.82645	3.28686	6.70193
1.15	1.17300	1.75614	3.10488	6.22755
1.20	1.15125	1.69444	2.94999	5.83565
1.25	1.13147	1.64036	2.81720	5.53080
1.30	1.11342	1.59172	2.70256	5.23381
1.35	1.09690	1.54874	2.60300	5.00059
1.40	1.08174	1.51020	2.51602	4.80133
1.45	1.06778	1.47562	2.43963	4.62990
1.50	1.05490	1.44444	2.37221	4.48148
1.55	1.04298	1.41623	2.31243	4.35221
1.60	1.03192	1.39062	2.25920	4.23901
1.65	1.02164	1.36731	2.2116	4.13936
1.70	1.01237	1.34602	2.16890	4.05123
1.75	1.00314	1.32653	2.13043	3.97292
1.80	.99479	1.30864	2.09567	3.90326
1.85	.98698	1.29218	2.06417	3.84048
1.90	.97965	1.27731	2.03552	3.76421
1.95	.97276	1.26258	2.00394	3.73345
2.00	.96628	1.25000	1.98554	3.68750
2.10	.95442	1.22676	1.94356	3.60777
2.20	.94384	1.20361	1.90802	3.54128
2.30	.93435	1.18903	1.87763	3.48527
2.40	.92581	1.17361	1.85147	3.43764
2.50	.91809	1.16030	1.82878	3.39680
2.60	.91108	1.14743	1.80898	3.36150
2.70	.90476	1.13717	1.79161	3.33079
2.80	.89888	1.1275	1.77628	3.30390
2.90	.89354	1.1189	1.76268	3.28022
3.00	.88863	1.11111	1.75057	3.25926
3.10	.88411	1.10406	1.73974	3.24060
3.20	.87994	1.09765	1.73001	3.22392
3.30	.87608	1.09182	1.72123	3.20895
3.40	.87250	1.08650	1.71329	3.19546
3.50	.86917	1.08163	1.70609	3.18325
3.60	.86607	1.07716	1.69953	3.17218
3.70	.86318	1.07334	1.69354	3.16210
3.80	.86047	1.06925	1.68806	3.15289
3.90	.85794	1.06574	1.68303	3.14446
4.00	.85557	1.06255	1.67840	3.13672
4.10	.85334	1.05940	1.67413	3.12959
4.20	.85124	1.05669	1.67019	3.12302
4.30	.84927	1.05406	1.66654	3.11694
4.40	.84741	1.05165	1.66315	3.11131
4.50	.84565	1.04930	1.66000	3.10608
4.60	.84399	1.04725	1.65707	3.10121
4.70	.84241	1.04527	1.65433	3.09668
4.80	.84093	1.04340	1.65178	3.09245
4.90	.83951	1.04165	1.64939	3.08850

TABLE A-1. (Concluded)

$\beta$	1st Noncentral Moment	2nd Noncentral Moment	3rd Noncentral Moment	4th Noncentral Moment
5.00	.83817	1.04056	1.64715	3.08480
5.10	.83698	1.03844	1.64505	3.08132
5.20	.83569	1.03698	1.64307	3.07866
5.30	.83453	1.03566	1.64122	3.07500
5.40	.83343	1.03429	1.63947	3.07211
5.50	.83238	1.03305	1.63782	3.06939
5.60	.83138	1.03188	1.63626	3.06682
5.70	.83043	1.03077	1.63479	3.06440
5.80	.82951	1.02972	1.63340	3.06210
5.90	.82864	1.02872	1.63208	3.05993
6.00	.82780	1.02777	1.63083	3.05787
6.50	.82416	1.02266	1.62545	3.04901
7.00	.82108	1.02040	1.62123	3.04206
7.50	.81856	1.01777	1.61785	3.03650
8.00	.81645	1.01562	1.61510	3.03197
8.50	.81466	1.01384	1.61284	3.02825
9.00	.81312	1.01234	1.61095	3.02514
9.50	.81179	1.01108	1.60936	3.02252
10.00	.81064	1.01003	1.60801	3.02029
11.00	.80873	1.00826	1.60585	3.01672
12.00	.80723	1.00694	1.60421	3.01401
13.00	.80603	1.00591	1.60295	3.01191
14.00	.80506	1.00510	1.60194	3.01026
15.00	.80425	1.00444	1.60114	3.00892
16.00	.80358	1.00390	1.60047	3.00781

TABLE A-2. CENTRAL MOMENTS WHEN  $\bar{X} = \bar{Y} = 0$ ,  $\beta = \frac{\sigma_X}{\sigma_Y}$

$\beta$	Mean	Standard Deviation	3rd Central Moment	4th Central Moment	Variance	Coefficient of Variation
1.00	1.25331423	.65513621	.17746061	.59779591	.42920345	1.91305901
1.05	1.22365533	.64207581	.16619825	.54576853	.40969704	1.91173500
1.10	1.19702367	.62735999	.15831244	.50712472	.39358056	1.90803318
1.15	1.17303224	.61661119	.15294793	.47786009	.38020936	1.90233693
1.20	1.15124623	.60751653	.14947566	.45574319	.36907633	1.89500397
1.25	1.13146663	.59981932	.14742497	.43901539	.35978322	1.88634576
1.30	1.113342165	.5933246	.14644635	.42639238	.35200781	1.87665099
1.35	1.09690489	.58778934	.14627209	.41692769	.34549631	1.86615308
1.40	1.08174239	.58312672	.14670467	.40985638	.34003678	1.85507256
1.45	1.06770309	.57919183	.14758101	.40477630	.33546318	1.84357414
1.50	1.054489762	.57587703	.14879188	.40101062	.33163436	1.83181053
1.55	1.04297671	.57308992	.15024087	.39850143	.32843205	1.81991807
1.60	1.03191924	.57075919	.15186247	.39683531	.32576600	1.80797656
1.65	1.02164412	.56681612	.15359822	.39596706	.32355177	1.79608855
1.70	1.01207277	.56722996	.15546662	.39567113	.32172707	1.78430025
1.75	1.00314380	.56589006	.15725645	.39579711	.32023156	1.77268320
1.80	.99474479	.56482232	.15911824	.39628961	.31902425	1.76125261
1.85	.98697911	.56396266	.16098469	.39704642	.31805390	1.75007877
1.90	.97964811	.56329044	.16282955	.39803049	.31729612	1.73915273
1.95	.97275989	.56277826	.16465011	.39917976	.31671937	1.72849585
2.00	.96626137	.56242335	.16642879	.40047100	.31629753	1.71812874
2.10	.95441664	.56119967	.16985065	.40343431	.31584059	1.69826296
2.20	.94384527	.56119467	.17308287	.40642062	.31578360	1.67958276
2.30	.93434789	.56216333	.17610082	.40961048	.31602764	1.66295759
2.40	.92584836	.56257116	.17891937	.41205100	.31648631	1.64567333
2.50	.91806624	.56312923	.18152553	.41597858	.31711453	1.63032956
2.60	.91118151	.56378775	.18394309	.41907877	.31785662	1.61600089
2.70	.90471412	.56452195	.18618126	.42209113	.31866502	1.60259742
2.80	.89887556	.56532473	.18824832	.42500138	.31956944	1.59007257
2.90	.89350571	.56612336	.191515634	.42781189	.32049559	1.57834116
3.00	.88663181	.56695696	.19192441	.43049830	.32144019	1.56737083
3.10	.88411438	.56779995	.19355501	.43308383	.32239679	1.55708781
3.20	.877991274	.56864171	.19506888	.43554813	.32335339	1.54744671
3.30	.876077953	.56948582	.19646691	.43791690	.32430007	1.53818365
3.40	.87249797	.57232530	.19776665	.44017038	.32524814	1.52987875
3.50	.86916878	.57111699	.19896911	.44233042	.32617462	1.52187519
3.60	.86660824	.57101288	.20038840	.44439149	.32708205	1.51434125
3.70	.86317521	.57268737	.20112604	.44636595	.32797082	1.50723635
3.80	.86047290	.57344653	.20209627	.44824320	.32883404	1.50054425
3.90	.85794113	.57417675	.20299575	.45004743	.32967894	1.49421084
4.00	.85556709	.57489257	.20383340	.45177597	.33050090	1.48822209
4.10	.85333763	.57558617	.20461488	.45343000	.33129444	1.48255408
4.20	.85124228	.57625690	.20534723	.45500723	.33207202	1.47719234
4.30	.84926756	.57692971	.20602934	.45652568	.33282401	1.47289966
4.40	.84740682	.57753846	.20666927	.45797604	.33355068	1.46727338
4.50	.84561860	.57814974	.20726748	.45937033	.33425713	1.46268092
4.60	.84396779	.57873952	.20782913	.4607518	.33493944	1.45832062
4.70	.84241477	.57931296	.20835343	.46199268	.33560297	1.45416296
4.80	.84092562	.57986459	.20884825	.46322273	.33624294	1.45021032
4.90	.83951326	.58034967	.20931280	.46444612	.33686285	1.44644192

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TABLE A-2. (Concluded)

$\beta$	Mean	Standard Deviation	3rd Central Moment	4th Central Moment	Variance	Coefficient of Variation
5.00	.83817238	.58591567	.24975000	.46554387	.33746302	1.44284689
5.10	.83689797	.56141623	.21016042	.46663973	.33854460	1.43941331
5.20	.83566581	.58189975	.210548C4	.46769302	.33860733	1.43613364
5.30	.83453169	.58236813	.21091230	.46874971	.33915263	1.43299684
5.40	.83343184	.58282148	.21125583	.46968901	.33968088	1.42999506
5.50	.83236298	.58326331	.21158014	.47063215	.34019259	1.42712089
5.60	.83130176	.58366500	.21188684	.47154099	.34068818	1.42436717
5.70	.83042537	.58419636	.21217640	.47241981	.34116855	1.42172666
5.80	.82951578	.58449522	.21245040	.47326684	.34163441	1.41919225
5.90	.82863702	.58487982	.21271001	.47408338	.34208441	1.41676457
6.00	.82779958	.58525369	.21295552	.47487426	.34252188	1.41442864
6.10	.82410407	.58695590	.21402635	.47844844	.34451723	1.40403062
7.00	.82117649	.58841965	.21482022	.48149280	.34623769	1.39539270
7.50	.81856269	.58966543	.21545924	.48410383	.34772891	1.38813449
8.00	.81645021	.59078727	.21596606	.48636900	.34902760	1.38196987
8.50	.81465651	.59175286	.21637096	.48834333	.35017137	1.37668383
9.00	.81311964	.59265275	.21669805	.49008016	.35117802	1.37211587
9.50	.81179123	.59335548	.21696344	.49161512	.35207072	1.36813639
10.00	.81063566	.59402432	.21718355	.49297012	.35286489	1.36465062
11.00	.80973520	.59515911	.21751107	.49524201	.35421437	1.35884704
12.00	.808724306	.59608186	.21773848	.49714623	.35531360	1.35423183
13.00	.80763469	.59683974	.21789947	.49866594	.35621774	1.35050426
14.00	.80655975	.59747291	.21801256	.49997215	.35697389	1.34744141
15.00	.80425465	.5981680	.21859423	.50104626	.35761213	1.34489214
16.00	.80358095	.59845969	.21815059	.50196481	.35815400	1.34274866

TABLE A-3. PROBABILITY DISTRIBUTION FUNCTION WHEN  $\bar{X} = \bar{Y} = 0$ ,

$$\beta = \frac{\sigma_X}{\sigma_Y}; F(s) = \int_0^s \beta t e^{-\frac{1}{4}(\beta^2+1)t^2} I_0 \left[ \frac{(\beta^2-1)t^2}{4} \right] dt$$

TABLE A-3. (Continued)

s	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
.00	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.05	.00250	.00242	.00274	.00287	.00299	.00312	.00324	.00337	.00349	.00361
.10	.00994	.01043	.01092	.01141	.01190	.01239	.01287	.01336	.01385	.01433
.15	.02219	.02327	.02435	.02542	.02649	.02754	.02862	.02968	.03073	.03178
.20	.03962	.04089	.04275	.04459	.04642	.04824	.05005	.05184	.05362	.05538
.25	.06013	.06294	.06573	.06858	.07121	.07390	.07657	.07920	.08180	.08457
.30	.08515	.08902	.09283	.09658	.10028	.10392	.10750	.11102	.11447	.11787
.35	.11366	.11849	.12352	.12831	.13300	.13760	.14209	.14648	.15076	.15494
.40	.14518	.15129	.15725	.16305	.16871	.17420	.17958	.18473	.18976	.19463
.45	.17923	.18642	.19340	.20016	.20670	.21301	.21909	.22494	.23060	.23602
.50	.21529	.22350	.23141	.23901	.24630	.25330	.25998	.26638	.27247	.27826
.55	.25787	.26198	.27068	.27899	.28689	.29440	.30153	.30829	.31447	.32070
.60	.29147	.30133	.31048	.31953	.32787	.33574	.34313	.35097	.35658	.36266
.65	.33043	.34109	.35091	.36012	.36874	.37678	.38427	.39123	.39770	.40369
.70	.36993	.38080	.39092	.40032	.40903	.41702	.42452	.43136	.43745	.44343
.75	.40897	.42008	.43032	.43974	.44839	.45631	.46355	.47015	.47616	.48163
.80	.44742	.45859	.46879	.47808	.48853	.49819	.50112	.50738	.51303	.51912
.85	.48498	.49605	.50608	.51509	.52322	.53051	.53705	.54291	.54815	.55283
.90	.52140	.53223	.54192	.55058	.55829	.56514	.57125	.57466	.58146	.58571
.95	.55649	.56695	.57622	.58442	.59166	.59804	.60365	.60860	.61276	.61678
1.00	.59010	.60009	.60886	.61659	.62326	.62912	.63425	.63873	.64265	.64608
1.05	.62211	.63156	.63977	.64649	.65307	.65942	.66305	.66708	.67059	.67365
1.10	.65245	.66130	.66892	.67587	.68109	.68594	.69011	.69371	.69763	.69956
1.15	.68016	.68930	.69631	.70278	.70738	.71173	.71546	.71867	.72145	.72385
1.20	.70800	.71557	.72196	.72737	.73196	.73586	.73910	.74203	.74449	.74662
1.25	.73320	.74012	.74592	.75050	.75490	.75837	.76133	.76385	.76603	.76771
1.30	.75673	.76301	.76824	.77261	.77627	.77936	.78197	.78421	.78613	.78780
1.35	.77862	.78429	.78899	.79268	.79613	.79987	.80119	.80316	.80487	.80635
1.40	.79893	.80403	.80822	.81148	.81457	.81699	.81903	.82078	.82229	.82360
1.45	.81773	.82229	.82602	.82909	.83164	.83374	.83559	.83713	.83847	.83963
1.50	.83550	.83914	.84245	.84517	.84742	.84931	.85091	.85228	.85346	.85499
1.55	.85107	.85467	.85760	.85999	.86198	.86345	.86504	.86627	.86737	.86823
1.60	.86576	.86895	.87153	.87364	.87539	.87684	.87811	.87918	.88011	.88092
1.65	.87923	.88204	.88432	.88618	.88772	.88902	.89012	.89107	.89169	.89261
1.70	.89155	.89403	.89603	.89767	.89903	.90017	.90115	.90199	.90271	.90335
1.75	.90281	.90499	.90675	.90819	.90936	.91039	.91128	.91199	.91263	.91320
1.80	.91307	.91498	.91652	.91779	.91889	.91973	.92049	.92114	.92171	.92221
1.85	.92240	.92907	.92542	.92653	.92746	.92824	.92891	.92949	.92999	.93093
1.90	.93084	.93233	.93351	.93449	.93530	.93599	.93657	.93708	.93753	.93791
1.95	.93851	.93981	.94085	.94170	.94241	.94302	.94353	.94398	.94437	.94471
2.00	.94545	.94657	.94748	.94873	.94985	.95030	.95084	.95023	.95057	.95087
2.10	.95732	.95817	.95884	.95943	.95991	.96032	.96067	.96097	.96123	.96197
2.20	.96684	.96793	.96805	.96848	.96885	.96916	.96933	.96966	.97000	.97041
2.30	.97452	.97501	.97540	.97573	.97601	.97624	.97649	.97662	.97677	.97691
2.40	.98057	.98093	.98123	.98147	.98168	.98186	.98201	.98214	.98226	.98236
2.50	.98531	.98558	.98580	.98598	.98614	.98627	.98639	.98649	.98657	.98668
2.60	.98899	.98919	.98936	.98949	.98961	.98970	.98979	.98993	.99079	.99093
2.70	.99183	.99197	.99209	.99219	.99228	.99236	.99241	.99246	.99251	.99255
2.80	.99398	.99409	.99418	.99425	.99431	.99436	.99443	.99448	.99458	.99452
2.90	.99561	.99569	.99578	.99580	.99585	.99589	.99592	.99595	.99597	.99600
3.00	.99683	.99688	.99693	.99697	.99700	.99703	.99705	.99707	.99709	.99711
3.10	.99773	.99777	.99780	.99783	.99785	.99787	.99789	.99790	.99791	.99793
3.20	.99839	.99842	.99844	.99846	.99847	.99849	.99850	.99851	.99852	.99853
3.30	.99987	.99989	.99990	.99992	.99993	.99994	.99995	.99995	.99996	.99996
3.40	.99921	.99922	.99924	.99924	.99925	.99926	.99927	.99927	.99927	.99928
3.50	.99946	.99947	.99947	.99948	.99948	.99949	.99949	.99950	.99950	.99950
3.60	.99963	.99963	.99964	.99964	.99965	.99965	.99965	.99965	.99966	.99966
3.70	.99975	.99975	.99976	.99976	.99976	.99976	.99976	.99977	.99977	.99977
3.80	.99983	.99983	.99984	.99984	.99984	.99984	.99984	.99984	.99984	.99984
3.90	.99989	.99989	.99989	.99989	.99989	.99989	.99989	.99990	.99990	.99990
4.00	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993
4.10	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99995	.99996	.99996
4.20	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997
4.30	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998
4.40	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
4.50	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
4.60	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
4.70	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.80	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.90	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
5.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

TABLE A-3. (Continued)

S	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
.00	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.05	.00374	.00386	.00199	.00411	.00423	.00436	.00448	.00460	.00473	.00485
.10	.1481	.01530	.01570	.01626	.01674	.01721	.01769	.01817	.01864	.01911
.15	.03282	.03386	.03489	.03592	.03694	.03796	.03897	.03997	.04097	.04196
.20	.05711	.05816	.06058	.06224	.06396	.06563	.06728	.06891	.07053	.07212
.25	.08692	.08940	.09186	.09479	.09668	.09704	.10135	.10343	.10568	.10808
.30	.12127	.12446	.12766	.13080	.13387	.13687	.13980	.14247	.14547	.14820
.35	.15901	.16298	.16684	.17060	.17424	.17779	.18123	.18456	.18779	.19092
.40	.19234	.20389	.20829	.21253	.21662	.22056	.22435	.22799	.23149	.23485
.45	.24122	.24671	.25098	.25555	.25992	.26409	.26804	.27195	.27545	.27888
.50	.28341	.29937	.29405	.29878	.30326	.30749	.31149	.31527	.31883	.32219
.55	.32639	.33174	.33177	.34170	.34494	.35009	.35398	.35762	.36103	.36420
.60	.36834	.37365	.37558	.38188	.38745	.39141	.39509	.39851	.40167	.40460
.65	.40923	.41436	.41908	.42344	.42746	.43115	.43455	.43768	.44056	.44320
.70	.44872	.45357	.45800	.46205	.46576	.46914	.47222	.47504	.47762	.47997
.75	.49659	.49119	.49518	.49889	.50225	.50530	.50806	.51058	.51294	.51494
.80	.52271	.52684	.53055	.53390	.53691	.53963	.54209	.54431	.54632	.54815
.85	.55701	.56075	.56409	.56709	.56977	.57218	.57435	.57631	.57807	.57968
.90	.58894	.59284	.59583	.59844	.60087	.60300	.60491	.60663	.60818	.60959
.95	.62017	.62316	.62581	.62817	.63027	.63215	.63383	.63535	.63672	.63796
1.00	.64917	.65175	.65410	.65618	.65809	.65969	.66118	.66252	.66373	.66639
1.05	.67613	.67848	.68074	.68259	.68423	.68570	.68701	.68819	.68927	.69024
1.10	.70192	.70470	.70584	.70747	.7092	.71021	.71138	.71243	.71359	.71425
1.15	.72725	.72779	.72942	.73076	.73219	.73329	.73433	.73526	.73611	.73667
1.20	.74448	.75011	.75155	.75282	.75394	.75499	.75591	.75674	.75750	.75819
1.25	.76956	.77100	.77228	.77342	.77443	.77534	.77616	.77671	.77759	.77820
1.30	.78926	.79054	.79168	.79248	.79359	.79440	.79513	.79588	.79641	.79696
1.35	.80764	.80878	.80978	.81068	.81149	.81221	.81287	.81351	.81400	.81450
1.40	.82475	.82576	.82666	.82776	.82817	.82882	.82941	.82994	.83042	.83087
1.45	.84065	.84155	.84235	.84306	.84370	.84428	.84480	.84528	.84571	.84611
1.50	.85539	.85619	.85690	.85754	.85811	.85863	.85909	.85952	.86071	.86026
1.55	.86974	.87075	.87038	.87095	.87146	.87192	.87233	.87271	.87304	.87338
1.60	.88144	.88277	.88283	.88313	.88379	.88420	.88457	.88491	.88522	.88550
1.65	.89124	.89190	.89130	.89175	.89216	.89252	.89285	.89315	.89443	.89668
1.70	.90391	.90441	.90485	.90525	.90561	.90594	.90623	.90659	.90674	.90697
1.75	.91370	.91419	.91453	.91489	.91520	.91549	.91575	.91599	.91621	.91641
1.80	.92265	.92314	.92339	.92370	.92393	.92424	.92447	.92468	.92488	.92505
1.85	.93362	.93116	.93147	.93175	.93200	.93223	.93243	.93262	.93279	.93295
1.90	.93826	.93856	.93884	.93918	.93930	.93950	.93969	.93985	.94000	.94014
1.95	.94552	.94629	.94553	.94574	.94594	.94612	.94628	.94642	.94656	.94668
2.00	.95114	.95138	.95159	.95178	.95195	.95211	.95225	.95238	.95250	.95261
2.10	.96167	.96186	.96212	.96217	.96230	.96242	.96253	.96243	.96272	.96280
2.20	.97021	.97134	.97046	.97058	.97068	.97077	.97085	.97073	.97100	.97107
2.30	.97713	.97714	.97723	.97732	.97740	.97747	.97753	.97759	.97764	.97769
2.40	.98245	.98253	.98261	.98267	.98273	.98278	.98283	.98288	.98292	.98295
2.50	.98677	.98675	.98683	.98698	.98692	.98696	.98700	.98703	.98706	.98709
2.60	.99003	.99008	.99012	.99016	.99019	.99022	.99025	.99027	.99029	.99031
2.70	.99259	.99242	.99265	.99268	.99271	.99273	.99275	.99276	.99278	.99280
2.80	.99454	.99457	.99459	.99461	.99463	.99464	.99466	.99467	.99468	.99469
2.90	.99602	.99603	.99605	.99606	.99608	.99609	.99610	.99611	.99612	.99612
3.00	.99717	.99713	.99714	.99715	.99716	.99717	.99718	.99718	.99719	.99720
3.10	.99794	.99794	.99795	.99796	.99797	.99797	.99798	.99798	.99799	.99799
3.20	.99853	.99851	.99855	.99855	.99856	.99856	.99856	.99857	.99857	.99857
3.30	.99827	.99827	.99828	.99828	.99828	.99829	.99829	.99829	.99829	.99829
3.40	.99928	.99928	.99929	.99929	.99929	.99930	.99930	.99930	.99930	.99930
3.50	.99951	.99951	.99951	.99951	.99951	.99951	.99951	.99951	.99952	.99952
3.60	.99966	.99966	.99966	.99966	.99967	.99967	.99967	.99967	.99967	.99967
3.70	.99977	.99977	.99977	.99977	.99977	.99977	.99977	.99977	.99978	.99978
3.80	.99985	.99985	.99985	.99985	.99985	.99985	.99985	.99985	.99985	.99985
3.90	.99990	.99990	.99990	.99990	.99990	.99990	.99990	.99990	.99990	.99990
4.00	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993	.99993
4.10	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996	.99996
4.20	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997	.99997
4.30	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99998	.99999
4.40	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
4.50	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999	.99999
4.60	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.70	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.80	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.90	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
5.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

ORIGINAL PAGE IS  
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TABLE A-3. (Continued)

TABLE A-3. (Concluded)

S	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9
.00	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.05	.00620	.00632	.00644	.00657	.00669	.00681	.00693	.00705	.00717	.00729
.10	.02421	.02467	.02512	.02557	.02602	.02646	.02691	.02735	.02779	.02823
.15	.05241	.05331	.05421	.05510	.05593	.05646	.05713	.05785	.05848	.06029
.20	.09345	.08982	.09117	.09250	.09391	.09509	.09636	.09761	.09884	.10005
.25	.15977	.13151	.13321	.12487	.12060	.13812	.11064	.14115	.14263	.14407
.30	.17733	.17593	.17779	.17364	.13192	.18315	.12493	.18645	.18802	.18353
.35	.21088	.22100	.22294	.22401	.22000	.22832	.21907	.22156	.22308	.23453
.40	.26352	.26597	.26733	.26000	.27078	.27239	.27200	.27535	.27673	.27804
.45	.30672	.30851	.31030	.31101	.31301	.31473	.31010	.31735	.31855	.31568
.50	.34913	.34977	.35125	.35260	.35326	.35520	.37026	.35745	.35847	.35944
.55	.36778	.38916	.39044	.38916	.39777	.39292	.37402	.39574	.39661	.35743
.60	.42556	.42675	.42785	.42293	.42985	.43075	.43158	.43239	.43313	.43383
.65	.46162	.46269	.46301	.46443	.46531	.46602	.46651	.46750	.46814	.46874
.70	.49667	.49695	.49773	.49355	.49927	.49929	.50057	.50117	.50172	.50225
.75	.52899	.52970	.52043	.53115	.53178	.53278	.53253	.53345	.53394	.53441
.80	.56045	.56112	.56173	.56235	.56291	.56343	.56322	.56439	.56492	.56523
.85	.59148	.59105	.59115	.59110	.59201	.59217	.59217	.59398	.59437	.59474
.90	.61814	.61937	.62017	.62046	.62103	.62150	.62139	.62226	.62281	.62234
.95	.64642	.64600	.64725	.64777	.64816	.64853	.64919	.64922	.64953	.64982
1.00	.67230	.67270	.67313	.67157	.67332	.67426	.67457	.67437	.67515	.67541
1.05	.68697	.69735	.69771	.69505	.69817	.69847	.69857	.69922	.69948	.69971
1.10	.72027	.72061	.72034	.72124	.72153	.72130	.72206	.72230	.72252	.72274
1.15	.74228	.74255	.74229	.74116	.74342	.74260	.74300	.74411	.74431	.74450
1.20	.75630	.76331	.76357	.76332	.76405	.76477	.76443	.76467	.76496	.76503
1.25	.78254	.78291	.78303	.78325	.78341	.78361	.78365	.78402	.78419	.78435
1.30	.80300	.80133	.80129	.80143	.80168	.80186	.80203	.80218	.80233	.80248
1.35	.81800	.81820	.81833	.81551	.81874	.81800	.81905	.81922	.81945	.81965
1.40	.83400	.83413	.83436	.83452	.83467	.83481	.83495	.83507	.83520	.83531
1.45	.84931	.84951	.84952	.84935	.84981	.84914	.84976	.84986	.84998	.85010
1.50	.86277	.86232	.86306	.86319	.86331	.86394	.86353	.86359	.86373	.86332
1.55	.87512	.87575	.87589	.87599	.87611	.87610	.87636	.87635	.87649	.87616
1.60	.89751	.89762	.89973	.89784	.89770	.89803	.89811	.89820	.89827	.89835
1.65	.90947	.89888	.89867	.89817	.89861	.89824	.89811	.89805	.89810	.89822
1.70	.90936	.90950	.90974	.90933	.90980	.90920	.90905	.90911	.90917	.90923
1.75	.91732	.91751	.91795	.91600	.91812	.91920	.91810	.91822	.91837	.91842
1.80	.92631	.92630	.92646	.92652	.92650	.92664	.92666	.92675	.92680	.92684
1.85	.92440	.93412	.93410	.93421	.93430	.93434	.93440	.93445	.93446	.93444
1.90	.94113	.94112	.94124	.94129	.94134	.94130	.94140	.94147	.94151	.94155
1.95	.94715	.94700	.94705	.94770	.94774	.94773	.94762	.94705	.94769	.94792
2.00	.95333	.95342	.95347	.95351	.95356	.95358	.95361	.95364	.95367	.95370
2.10	.96340	.96344	.96347	.96350	.96353	.96356	.96358	.96361	.96361	.96365
2.20	.97152	.97155	.97153	.97165	.97152	.97166	.97166	.97168	.97170	.97172
2.30	.97204	.97200	.97301	.97210	.97812	.97312	.97815	.97810	.97818	.97819
2.40	.98322	.99323	.99325	.99326	.99327	.99329	.99330	.99331	.99332	.99333
2.50	.99716	.99710	.99731	.99732	.99733	.99734	.99725	.99736	.99737	.99727
2.60	.99940	.99947	.99949	.99949	.99950	.99950	.99951	.99951	.99952	.99952
2.70	.99951	.99951	.99922	.99930	.99923	.99923	.99924	.99924	.99925	.99926
2.80	.99477	.99473	.99473	.99473	.99473	.99479	.99480	.99430	.99430	.99481
2.90	.99118	.99115	.99112	.99118	.99018	.99012	.99010	.99021	.99021	.99021
3.00	.99724	.99724	.99724	.99728	.99725	.99725	.99725	.99725	.99725	.99726
3.10	.99902	.99911	.99911	.99911	.99912	.99912	.99913	.99913	.99913	.99913
3.20	.99959	.99950	.99930	.99960	.99960	.99960	.99960	.99960	.99955	.99966
3.30	.99901	.99901	.99901	.99901	.99901	.99902	.99901	.99902	.99903	.99905
3.40	.99731	.99931	.99931	.99931	.99931	.99932	.99931	.99935	.99935	.99925
3.50	.99952	.99951	.99951	.99954	.99935	.99919	.99900	.99872	.99854	.99826
3.60	.99957	.99957	.99950	.99943	.99933	.99920	.99901	.99879	.99864	.99925
3.70	.99976	.99976	.99965	.99940	.99935	.99919	.99900	.99880	.99854	.99825
3.80	.99976	.99976	.99961	.99961	.99963	.99930	.99920	.99879	.99854	.99825
3.90	.99977	.99977	.99968	.99968	.99948	.99935	.99922	.99911	.99901	.99925
4.00	.99976	.99976	.99970	.99966	.99947	.99936	.99920	.99901	.99880	.99854
4.10	.99976	.99976	.99978	.99964	.99951	.99919	.99911	.99875	.99855	.99825
4.20	.99976	.99976	.99973	.99950	.99935	.99919	.99901	.99879	.99854	.99825
4.30	.99976	.99976	.99976	.99950	.99937	.99920	.99911	.99880	.99855	.99826
4.40	.99976	.99976	.99970	.99950	.99943	.99913	.99901	.99879	.99854	.99826
4.50	.99977	.99977	.99976	.99951	.99948	.99913	.99901	.99879	.99854	.99826
4.60	.99976	.99976	.99970	.99953	.99943	.99916	.99902	.99879	.99854	.99826
4.70	.99976	.99976	.99976	.99950	.99945	.99916	.99902	.99879	.99853	.99825
4.80	.99977	.99976	.99970	.99953	.99943	.99916	.99902	.99879	.99854	.99825
4.90	.99976	.99976	.99975	.99951	.99946	.99915	.99902	.99879	.99854	.99825
5.00	.99976	.99976	.99973	.99951	.99941	.99910	.99901	.99880	.99854	.99826

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TABLE A-4. NONCENTRAL MOMENTS WHEN  $\sigma_X = \sigma_Y = \sigma$ ,  $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$

SIGMA X=SIGMA Y

$\lambda$	1st Noncentral Moment	2nd Noncentral Moment	3rd Noncentral Moment	4th Noncentral Moment
.00	1.25331	2.00000	3.75994	8.00000
.05	1.25410	2.00250	3.76699	8.02001
.10	1.25545	2.01000	3.78816	8.08010
.15	1.26035	2.02250	3.82348	8.18051
.20	1.26582	2.04000	3.87302	8.32160
.25	1.27282	2.06250	3.93688	8.50391
.30	1.28136	2.09000	4.01516	8.72910
.35	1.29141	2.12250	4.10802	8.99500
.40	1.30295	2.16000	4.21563	9.30560
.45	1.31597	2.20250	4.33817	9.66101
.50	1.33045	2.25000	4.47587	10.06250
.55	1.34635	2.30250	4.62897	10.51151
.60	1.36365	2.36000	4.73774	11.00960
.65	1.38232	2.42250	4.98247	11.55850
.70	1.40233	2.49000	5.18347	12.16010
.75	1.42364	2.56250	5.40108	12.81640
.80	1.44623	2.64000	5.63566	13.52960
.85	1.47005	2.72250	5.88758	14.30200
.90	1.49508	2.81000	6.15724	15.13610
.95	1.52126	2.90250	6.44505	16.03450
1.00	1.54857	3.00000	6.75147	17.00000
1.05	1.57697	3.10250	7.07683	18.03550
1.10	1.60641	3.21000	7.42192	19.14410
1.15	1.63686	3.32250	7.78693	20.32900
1.20	1.66827	3.44000	8.17245	21.59359
1.25	1.70062	3.56250	8.57901	22.94140
1.30	1.73386	3.69000	9.00714	24.37609
1.35	1.76794	3.82250	9.45740	25.90150
1.40	1.80284	3.96000	9.93035	27.52159
1.45	1.83852	4.10250	10.42656	29.24050
1.50	1.87494	4.25000	10.94661	31.06249
1.55	1.91205	4.40250	11.49112	32.99200
1.60	1.94984	4.56000	12.06088	35.03353
1.65	1.98827	4.72250	12.65592	37.19200
1.70	2.02723	4.89000	13.27746	39.47209
1.75	2.06689	5.06250	13.92594	41.87690
1.80	2.10702	5.24000	14.60201	44.41759
1.85	2.14767	5.42250	15.30632	47.09350
1.90	2.18979	5.61000	16.03953	49.91203
1.95	2.23037	5.80250	16.80231	52.87899
2.00	2.27239	6.00000	17.53532	55.99999
2.05	2.31480	6.20250	18.41926	59.28099
2.10	2.35759	6.41000	19.27480	62.72903
2.15	2.40074	6.62250	20.16263	66.34749
2.20	2.44423	6.84000	21.03346	70.14559
2.25	2.48804	7.06500	22.03797	74.12689
2.30	2.53215	7.29000	23.02687	78.30407
2.35	2.57653	7.52250	24.05087	82.67797
2.40	2.62119	7.76000	25.11068	87.25757
2.45	2.66607	8.00250	26.20700	92.04997

TABLE A-4. (Continued)

$\lambda$	1st Noncentral Moment	2nd Noncentral Moment	3rd Noncentral Moment	4th Noncentral Moment
2.50	2.71120	8.25000	27.34056	97.06246
2.55	2.75655	8.50250	28.51208	102.30246
2.60	2.80210	8.76000	29.72227	107.77755
2.65	2.84784	9.02250	30.97185	113.49545
2.70	2.89377	9.29000	32.26156	119.46404
2.75	2.93986	9.56250	33.59212	125.69134
2.80	2.98612	9.84000	34.96426	132.18553
2.85	3.03253	10.12250	36.37871	138.95493
2.90	3.07908	10.41000	37.83620	146.00801
2.95	3.12577	10.70250	39.33747	153.35341
3.00	3.17258	11.00000	40.89324	160.93990
3.05	3.21951	11.30250	42.47426	168.95639
3.10	3.26655	11.61000	44.11127	177.23199
3.15	3.31370	11.92250	45.79489	185.83589
3.20	3.36095	12.23999	47.52617	194.77749
3.25	3.40830	12.56249	49.30554	204.06626
3.30	3.45574	12.88999	51.13386	213.71194
3.35	3.50326	13.22249	53.01105	223.72434
3.40	3.55086	13.55999	54.94026	234.11342
3.45	3.59854	13.90249	56.91983	244.88932
3.50	3.64630	14.24999	58.95131	256.06230
3.55	3.69412	14.60249	61.03543	267.64279
3.60	3.74201	14.35999	63.17293	279.64137
3.65	3.78997	15.32249	65.36458	292.06876
3.70	3.83798	15.63999	67.61110	304.93583
3.75	3.88605	16.06249	69.91325	318.25364
3.80	3.93413	16.43999	72.27176	332.03332
3.85	3.98236	16.82249	74.68738	346.28618
3.90	4.03059	17.20999	77.16086	361.02377
3.95	4.07887	17.60249	79.69295	376.25767
4.00	4.12719	17.33999	82.28438	391.99966
4.05	4.17556	18.40249	84.93590	408.26162
4.10	4.22397	18.80399	87.64827	425.05570
4.15	4.27242	19.22249	90.42221	442.39404
4.20	4.32092	19.63999	93.25851	460.23912
4.25	4.36944	20.06249	96.15788	478.75341
4.30	4.41801	20.48999	99.12107	497.79953
4.35	4.46661	20.92248	102.14884	517.44042
4.40	4.51524	21.35998	105.24194	537.68395
4.45	4.56391	21.80248	108.40110	558.55831
4.50	4.61260	22.24998	111.62709	580.06180
4.55	4.66133	22.70248	114.92063	602.21272
4.60	4.71009	23.15998	118.28248	625.02479
4.65	4.75987	23.62248	121.71341	648.51163
4.70	4.80768	24.08993	125.21414	672.68713
4.75	4.85652	24.56248	128.78542	697.56542
4.80	4.90538	25.03993	132.42802	723.16050
4.85	4.95427	25.52248	136.14267	749.48694
4.90	5.00319	26.00997	139.93011	776.55893
4.95	5.05211	26.50247	143.79111	804.39124
5.00	5.10107	26.99997	147.72640	832.99871
5.05	5.15004	27.50247	151.73673	862.39611
5.10	5.19904	28.00997	155.82287	892.59863
5.15	5.24806	28.52247	159.98556	923.62154
5.20	5.29710	29.03997	164.22553	955.47997
5.25	5.34615	29.56247	168.54354	988.18977

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TABLE A-4. (Continued)

$\lambda$	1st Noncentral Moment	2nd Noncentral Moment	3rd Noncentral Moment	4th Noncentral Moment
5.30	5.39523	30.03997	172.94035	1021.76633
5.35	5.44432	30.62247	177.41669	1056.22560
5.40	5.49343	31.15097	181.97332	1091.53362
5.45	5.54256	31.70246	186.61089	1127.85646
5.50	5.59170	32.24996	191.33043	1165.06032
5.55	5.64066	32.80246	196.13244	1203.21175
5.60	5.69003	33.35936	201.01770	1242.32729
5.65	5.73922	33.92246	205.98700	1282.42351
5.70	5.78842	34.48996	211.04109	1323.51752
5.75	5.83764	35.06246	216.18072	1365.62621
5.80	5.88687	35.63996	221.40662	1408.76682
5.85	5.93612	36.22246	226.71955	1452.95663
5.90	5.98533	36.80995	232.12024	1498.21301
5.95	6.03465	37.40245	237.60949	1544.55383
6.00	6.08393	37.99395	243.18801	1591.99667
6.05	6.13323	38.60245	248.85653	1640.55952
6.10	6.18254	39.20995	254.61587	1690.26053
6.15	6.23186	39.82245	260.46669	1741.11766
6.20	6.28119	40.43995	266.40982	1793.14975
6.25	6.33053	41.06244	272.44595	1846.37482
6.30	6.37981	41.63994	278.57586	1900.81183
6.35	6.42924	42.32244	284.80029	1956.47958
6.40	6.47862	42.95994	291.12000	2013.39711
6.45	6.52800	43.60244	297.53573	2071.58334
6.50	6.57739	44.24994	304.04824	2131.05771
6.55	6.62679	44.90243	310.65824	2191.83914
6.60	6.67620	45.55993	317.36654	2253.94830
6.65	6.72562	46.22243	324.17382	2317.40390
6.70	6.77505	46.88993	331.08093	2382.22641
6.75	6.82449	47.56243	338.08851	2448.43561
6.80	6.87393	48.23993	345.13742	2516.05176
6.85	6.92339	48.92243	352.40828	2585.09475
6.90	6.97285	49.60993	359.72194	2655.58554
6.95	7.02232	50.30242	367.13910	2727.54453
7.00	7.07180	50.99992	374.66054	2800.99301
7.05	7.12123	51.70242	382.28696	2875.95111
7.10	7.17078	52.40991	390.01916	2952.44061
7.15	7.22023	53.12242	397.85783	3030.48227
7.20	7.26973	53.83991	405.80393	3110.09775
7.25	7.31930	54.56242	413.85798	3191.30841
7.30	7.36882	55.23991	422.02071	3274.13542
7.35	7.41834	56.02241	430.29304	3358.60175
7.40	7.46783	56.75990	438.67556	3444.72838
7.45	7.51742	57.50240	447.16915	3532.53799
7.50	7.56696	58.24990	455.77448	3622.05270
7.55	7.61652	59.00240	464.49231	3713.29510
7.60	7.66607	59.75990	473.32346	3806.28714
7.65	7.71564	60.52239	482.26854	3901.05209
7.70	7.76521	61.28989	491.32847	3997.61276
7.75	7.81478	62.06239	500.50383	4095.99289
7.80	7.86436	62.83989	509.73549	4196.21405
7.85	7.91395	63.62239	519.20416	4298.30060
7.90	7.96354	64.40989	528.73052	4402.27490
7.95	8.01314	65.20233	538.37553	4508.16302
8.00	8.06274	65.99983	548.13974	4615.98657
8.05	8.11235	66.80238	558.02396	4725.77045

TABLE A-4. (Concluded)

$\lambda$	1st Noncentral Moment	2nd Noncentral Moment	3rd Noncentral Moment	4th Noncentral Moment
8.10	8.16196	67.60987	568.02891	4837.53773
8.15	8.21158	68.42237	578.15538	4951.31390
8.20	8.26120	69.23387	588.40417	5067.12286
8.25	8.31082	70.06237	598.77589	5184.98853
8.30	8.36045	70.99987	609.27133	530.93604
8.35	8.41009	71.72237	619.89144	5426.99078
8.40	8.45973	72.55996	630.63670	5551.17639
8.45	8.50937	73.40230	641.50806	5677.51996
8.50	8.55902	74.24996	652.50604	5806.04486
8.55	8.60867	75.10236	663.63159	5936.77699
8.60	8.65833	75.95995	674.89546	6069.74341
8.65	8.70799	76.82235	686.26826	6204.96759
8.70	8.75765	77.68985	697.78084	6342.47620
8.75	8.80732	78.56234	709.42397	6482.29669
8.80	8.85699	79.43984	721.19830	6624.45245
8.85	8.90667	80.32235	733.10461	6768.97345
8.90	8.95635	81.20983	745.14373	6915.88245
8.95	9.00603	82.10233	757.31641	7065.20825
9.00	9.05572	82.99984	769.62322	7216.97705
9.05	9.10541	83.90233	782.06514	7371.21698
9.10	9.15510	84.80982	794.64277	7527.95111
9.15	9.20480	85.72232	807.35692	7687.21185
9.20	9.25450	86.63983	820.20857	7849.02594
9.25	9.30420	87.56231	833.19781	8013.41498
9.30	9.35391	88.48981	846.32592	8180.91284
9.35	9.40362	89.42231	859.59369	8350.04480
9.40	9.45333	90.35980	873.00181	8522.34113
9.45	9.50305	91.30231	886.55043	8697.32654
9.50	9.55276	92.24980	900.24110	8875.03210
9.55	9.60249	93.20229	914.07435	9055.48535
9.60	9.65221	94.15973	929.05081	9238.71449
9.65	9.70194	95.12229	942.17123	9424.74707
9.70	9.75167	96.08971	956.43680	9613.61487
9.75	9.80140	97.06228	970.84727	9805.34485
9.80	9.85114	98.03979	985.40424	9999.96692
9.85	9.90088	99.02228	1000.10821	10197.51013
9.90	9.95062	100.00977	1014.95990	10398.00231
9.95	10.00036	101.00227	1029.96008	10601.47766
10.00	10.05011	101.99973	1045.10366	10807.96179

TABLE A-5. CENTRAL MOMENTS WHEN  $\sigma_X = \sigma_Y = \sigma$ ,  $\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}$

SIGMA X=SIGMA Y						
$\lambda$	Mean	Standard Deviation	3rd Central Moment	4th Central Moment	Variance	Coefficient of Variation
.00	1.25331	.65514	.17746	.59773	.42920	1.91306
.05	1.25010	.65355	.17773	.59929	.42970	1.91306
.10	1.25645	.65677	.17878	.60370	.43134	1.91307
.15	1.26035	.65879	.18001	.61111	.43801	1.91313
.20	1.26502	.66160	.18264	.62149	.43771	1.91329
.25	1.27282	.66515	.18542	.63464	.44263	1.91353
.30	1.28136	.66942	.18871	.65053	.44813	1.91412
.35	1.29141	.67437	.19243	.66907	.45427	1.91493
.40	1.30295	.67954	.19650	.69015	.46232	1.91629
.45	1.31597	.68609	.20005	.71369	.47021	1.91803
.50	1.33045	.69276	.20539	.73941	.47991	1.92052
.55	1.34635	.69989	.21000	.76734	.48995	1.92366
.60	1.36365	.70743	.21463	.79726	.50046	1.92766
.65	1.38232	.71533	.21916	.82906	.51170	1.93292
.70	1.40233	.72352	.22351	.86258	.52348	1.93821
.75	1.42364	.73196	.22760	.89769	.53570	1.94502
.80	1.44623	.74055	.23133	.93426	.54842	1.95291
.85	1.47005	.74929	.23466	.97215	.56109	1.96132
.90	1.49508	.75812	.23750	1.01123	.57470	1.97209
.95	1.52126	.76698	.23980	1.05138	.58826	1.98344
1.00	1.54857	.77584	.24151	1.09251	.60192	1.99600
1.05	1.57697	.78465	.24260	1.13499	.61567	2.00577
1.10	1.60641	.79337	.24305	1.17719	.62945	2.02477
1.15	1.63686	.80199	.24283	1.22053	.64319	2.04093
1.20	1.66827	.81047	.24194	1.26444	.65686	2.05841
1.25	1.70062	.81877	.24039	1.30880	.67039	2.07709
1.30	1.73386	.82689	.23820	1.35355	.68374	2.09685
1.35	1.76794	.83479	.23536	1.39850	.69698	2.11703
1.40	1.80284	.84247	.23193	1.44367	.70976	2.13995
1.45	1.83852	.84991	.22795	1.48891	.72230	2.16320
1.50	1.87496	.85710	.22343	1.53419	.73462	2.18754
1.55	1.91265	.86403	.21843	1.57940	.74655	2.21293
1.60	1.94984	.87070	.21300	1.62441	.75811	2.23941
1.65	1.98827	.87710	.20720	1.66918	.76930	2.26683
1.70	2.02729	.88323	.20167	1.71364	.78009	2.29532
1.75	2.06689	.88909	.19467	1.75765	.79040	2.32472
1.80	2.10702	.89469	.18804	1.80018	.80046	2.35569
1.85	2.14767	.90002	.18125	1.84410	.81003	2.38625
1.90	2.18879	.90509	.17430	1.88630	.81919	2.41031
1.95	2.23037	.90991	.16737	1.92788	.82793	2.45121
2.00	2.27238	.91443	.16036	1.96864	.83627	2.48499
2.05	2.31480	.91881	.15337	2.00848	.84421	2.51234
2.10	2.35759	.92291	.14643	2.04742	.85176	2.55452
2.15	2.40074	.92678	.13959	2.08236	.85892	2.59001
2.20	2.44423	.93044	.13267	2.12226	.86572	2.62697
2.25	2.48804	.93389	.12629	2.15810	.87215	2.66417
2.30	2.53215	.93714	.11982	2.19282	.87820	2.70198
2.35	2.57653	.94021	.11367	2.22640	.88399	2.74039
2.40	2.62118	.94309	.10767	2.25883	.88902	2.77934
2.45	2.66607	.94581	.10188	.29005	.89455	2.81803

TABLE A-5. (Continued)

$\lambda$	Mean	Standard Deviation	3rd Central Moment	4th Central Moment	Variance	Coefficient of Variation
2.50	2.71120	.94936	.09632	2.32011	.89939	2.85883
2.55	2.75655	.95076	.09100	2.34897	.90395	2.89931
2.60	2.80210	.95302	.08593	2.37663	.90824	2.94024
2.65	2.84784	.95510	.08103	2.40314	.91229	2.98160
2.70	2.89377	.95713	.07649	2.42849	.91610	3.02387
2.75	2.93986	.95901	.07214	2.45268	.91969	3.06557
2.80	2.98612	.96077	.06801	2.47577	.92307	3.10806
2.85	3.03253	.96242	.06412	2.49775	.92626	3.15093
2.90	3.07908	.96398	.06046	2.51867	.92926	3.19413
2.95	3.12577	.96545	.05700	2.53855	.93209	3.23764
3.00	3.17258	.96683	.05376	2.55745	.93475	3.28143
3.05	3.21951	.96813	.05072	2.57534	.93727	3.32551
3.10	3.26655	.96935	.04786	2.59235	.93964	3.36984
3.15	3.31370	.97051	.04519	2.60850	.94188	3.41881
3.20	3.36095	.97159	.04269	2.62381	.94399	3.45941
3.25	3.40830	.97262	.04034	2.63828	.94600	3.50423
3.30	3.45570	.97360	.03815	2.65202	.94789	3.54946
3.35	3.50326	.97451	.03611	2.66499	.94988	3.59483
3.40	3.55086	.97538	.03421	2.67722	.95137	3.64048
3.45	3.59854	.97621	.03240	2.68901	.95299	3.68624
3.50	3.64630	.97693	.03074	2.70005	.95451	3.73217
3.55	3.69412	.97773	.02918	2.71051	.95536	3.77825
3.60	3.74201	.97844	.02772	2.72046	.95734	3.82448
3.65	3.78997	.97911	.02636	2.72981	.95865	3.87184
3.70	3.83798	.97974	.02508	2.73889	.95990	3.91733
3.75	3.88605	.98035	.02390	2.74715	.96107	3.96390
3.80	3.93418	.98093	.02274	2.75536	.96222	4.01066
3.85	3.98236	.98148	.02172	2.76297	.96331	4.05750
3.90	4.03059	.98201	.02075	2.77020	.96434	4.10444
3.95	4.07887	.99251	.01983	2.77711	.96533	4.15107
4.00	4.12719	.98299	.01895	2.78382	.96628	4.19860
4.05	4.17556	.98345	.01815	2.79001	.96718	4.24581
4.10	4.22397	.98389	.01739	2.79593	.96805	4.29312
4.15	4.27202	.98432	.01663	2.80139	.96890	4.34043
4.20	4.32092	.98472	.01600	2.80705	.96960	4.38795
4.25	4.36944	.99511	.01530	2.81239	.97042	4.43540
4.30	4.41801	.98543	.01471	2.81763	.97119	4.48307
4.35	4.46661	.98585	.01416	2.82225	.97199	4.53070
4.40	4.51524	.98619	.01363	2.82666	.97257	4.57847
4.45	4.56391	.99653	.01303	2.83147	.97324	4.62624
4.50	4.61260	.98685	.01262	2.83539	.97396	4.67409
4.55	4.66133	.99715	.01217	2.83936	.97467	4.72199
4.60	4.71009	.99745	.01169	2.84370	.97506	4.76993
4.65	4.75887	.99770	.01132	2.84695	.97563	4.81793
4.70	4.80760	.99802	.01091	2.85071	.97617	4.86600
4.75	4.85652	.99828	.01050	2.85452	.97671	4.91403
4.80	4.90538	.99854	.01020	2.85730	.97721	4.96225
4.85	4.95427	.99879	.00985	2.86070	.97770	5.01004
4.90	5.00318	.99903	.00952	2.86375	.97818	5.05866
4.95	5.05211	.99927	.00920	2.86668	.97865	5.10633
5.00	5.10107	.99949	.00887	2.86998	.97910	5.15523
5.05	5.15004	.99971	.00861	2.87259	.97953	5.20359
5.10	5.19904	.99992	.00835	2.87524	.97994	5.25198
5.15	5.24806	.99013	.00804	2.87830	.98035	5.30033
5.20	5.29710	.99037	.00783	2.88040	.98074	5.34885
5.25	5.30615	.99052	.00759	2.88303	.98112	5.39733

ORIGINAL TABLE IS  
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TABLE A-5. (Continued)

$\lambda$	Mean	Standard Deviation	3rd Central Moment	4th Central Moment	Variance	Coefficient of Variation
5.30	.39523	.99070	.00739	2.88498	.98149	5.44585
5.35	5.44932	.99089	.00714	2.88751	.98185	5.49440
5.40	5.49343	.99106	.00691	2.89001	.98221	5.54297
5.45	5.54256	.99123	.00674	2.89182	.98254	5.59158
5.50	5.59170	.99140	.00655	2.89398	.98287	5.64022
5.55	5.64086	.99156	.00636	2.89575	.98319	5.68887
5.60	5.69003	.99172	.00618	2.89786	.98350	5.73756
5.65	5.73922	.99187	.00595	2.90030	.98381	5.78626
5.70	5.78842	.99202	.00582	2.90198	.98410	5.83501
5.75	5.83764	.99216	.00560	2.90408	.98439	5.88373
5.80	5.88687	.99230	.00552	2.90518	.98466	5.93255
5.85	5.93612	.99244	.00529	2.90787	.98493	5.98135
5.90	5.98538	.99257	.00517	2.90909	.98520	6.03017
5.95	6.03465	.99270	.00507	2.91049	.98545	6.07904
6.00	6.08393	.99283	.00493	2.91187	.98570	6.12790
6.05	6.13323	.99295	.00478	2.91376	.98594	6.17679
6.10	6.18254	.99307	.00460	2.91473	.98618	6.22570
6.15	6.23186	.99318	.00451	2.91681	.98642	6.27462
6.20	6.28119	.99330	.00443	2.91785	.98664	6.32357
6.25	6.33053	.99341	.00435	2.91901	.98685	6.37255
6.30	6.37988	.99351	.00428	2.91998	.98707	6.42159
6.35	6.42924	.99362	.00409	2.92194	.98728	6.47051
6.40	6.47862	.99372	.00402	2.92316	.98748	6.51955
6.45	6.52800	.99381	.00381	2.92566	.98769	6.56856
6.50	6.57739	.99392	.00383	2.92560	.98787	6.61764
6.55	6.62679	.99402	.00372	2.92645	.98807	6.66668
6.60	6.67620	.99411	.00366	2.92743	.98825	6.71577
6.65	6.72562	.99420	.00353	2.92920	.98843	6.76485
6.70	6.77505	.99429	.00345	2.92999	.98861	6.81396
6.75	6.82449	.99438	.00330	2.93225	.98879	6.86306
6.80	6.87393	.99446	.00322	2.93323	.98896	6.91221
6.85	6.92339	.99455	.00314	2.93433	.98912	6.96135
6.90	6.97285	.99463	.00306	2.93500	.98929	7.01050
6.95	7.02232	.99471	.00302	2.93536	.98944	7.05968
7.00	7.07180	.99478	.00307	2.93530	.98959	7.10891
7.05	7.12128	.99486	.00283	2.93781	.98975	7.15807
7.10	7.17078	.99493	.00290	2.93811	.98989	7.20731
7.15	7.22028	.99501	.00272	2.94019	.99004	7.25650
7.20	7.26978	.99507	.00280	2.93909	.99017	7.30577
7.25	7.31930	.99515	.00257	2.94165	.99032	7.35497
7.30	7.36882	.99522	.00258	2.94189	.99045	7.40424
7.35	7.41834	.99529	.00250	2.94275	.99059	7.45350
7.40	7.46788	.99535	.00242	2.94385	.99072	7.50277
7.45	7.51742	.99542	.00233	2.94470	.99085	7.55204
7.50	7.56696	.99548	.00227	2.94617	.99098	7.60133
7.55	7.61652	.99553	.00237	2.94507	.99109	7.65068
7.60	7.66607	.99560	.00217	2.94666	.99122	7.69995
7.65	7.71564	.99565	.00236	2.94556	.99132	7.74935
7.70	7.76521	.99571	.00218	2.94604	.99145	7.79863
7.75	7.81478	.99577	.00211	2.94946	.99155	7.84800
7.80	7.86436	.99583	.00194	2.95142	.99167	7.89731
7.85	7.91395	.99589	.00176	2.95325	.99179	7.94662
7.90	7.96354	.99593	.00195	2.95154	.99188	7.99607
7.95	8.01314	.99599	.00172	2.95325	.99200	8.04537
8.00	8.06274	.99604	.00168	2.95179	.99209	8.09482
8.05	8.11235	.99609	.00168	2.95386	.99220	8.14417

TABLE A-5. (Concluded)

$\lambda$	Mean	Standard Deviation	3rd Central Moment	4th Central Moment	Variance	Coefficient of Variation
8.10	8.16196	.99615	.00159	2.95520	.99231	8.19354
8.15	8.21158	.99619	.00172	2.95398	.99239	8.24300
8.20	8.26120	.99624	.00156	2.95581	.99249	8.29238
8.25	8.31082	.99628	.00163	2.95569	.99258	8.34184
8.30	8.36045	.99633	.00131	2.96167	.99268	8.39123
8.35	8.41009	.99639	.00119	2.96130	.99278	8.44060
8.40	8.45973	.99642	.00148	2.95752	.99285	8.49014
8.45	8.50937	.99647	.00125	2.95947	.99295	8.53951
8.50	8.55902	.99650	.00137	2.95984	.99302	8.58904
8.55	8.60867	.99656	.00099	2.96533	.99312	8.63842
8.60	8.65833	.99653	.00130	2.96045	.99319	8.68794
8.65	8.70799	.99663	.00128	2.96094	.99327	8.73743
8.70	8.75765	.99668	.00099	2.96558	.99336	8.78686
8.75	8.80732	.99671	.00113	2.96216	.99344	8.83636
8.80	8.85699	.99675	.00122	2.96069	.99350	8.88590
8.85	8.90667	.99679	.00082	2.96997	.99359	8.93535
8.90	8.95635	.99682	.00105	2.96411	.99366	8.98488
8.95	9.00603	.99686	.00113	2.96289	.99373	9.03441
9.00	9.05572	.99691	.00052	2.97339	.99383	9.08379
9.05	9.10541	.99693	.00113	2.96460	.99386	9.13349
9.10	9.15510	.99697	.00101	2.96338	.99394	9.18295
9.15	9.20480	.99701	.00066	2.96973	.99403	9.23241
9.20	9.25450	.99702	.00127	2.96118	.99405	9.28114
9.25	9.30420	.99707	.00089	2.96411	.99415	9.33154
9.30	9.35391	.99710	.00070	2.96948	.99422	9.38108
9.35	9.40362	.99713	.00108	2.96118	.99427	9.43070
9.40	9.45333	.99716	.00101	2.96240	.99437	9.48025
9.45	9.50305	.99720	.00046	2.97339	.99441	9.52970
9.50	9.55276	.99724	.00031	2.97607	.99448	9.57922
9.55	9.60249	.99725	.00082	2.96777	.99451	9.62896
9.60	9.65221	.99729	.00072	2.9782	.99455	9.67845
9.65	9.70194	.99733	.00012	2.97900	.99467	9.72780
9.70	9.75167	.99735	.00056	2.96924	.99470	9.77762
9.75	9.80140	.99737	.00070	2.96655	.99476	9.82721
9.80	9.85114	.99742	-.00009	2.95218	.99484	9.87665
9.85	9.90088	.99743	.00040	2.97314	.99487	9.92637
9.90	9.95062	.99746	.00047	2.97290	.99492	9.97600
9.95	10.00036	.99749	.00029	2.97778	.99498	10.02555
10.00	10.05011	.99753	-.00020	2.98340	.99506	10.07504

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TABLE A-6. PROBABILITY DISTRIBUTION WHEN  $\sigma_X = \sigma_Y = \sigma$ ,

$$\lambda^2 = \frac{\bar{X}^2 + \bar{Y}^2}{\sigma^2}; F(s) = \int_0^s t e^{-\frac{1}{2}(t^2 + \lambda^2)} I_0(\lambda t) dt$$

GENERALIZED RAYLEIGH DISTRIBUTION FUNCTION(2)  
H(t)  
LAMBDA

s	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.1	.00499	.00496	.00489	.00477	.00460	.00440	.00417	.00391	.00362	.00333
.2	.01980	.01970	.01941	.01894	.01829	.01751	.01651	.01554	.01442	.01326
.3	.04400	.04379	.04315	.04211	.04069	.03894	.03690	.03463	.03218	.02961
.4	.07688	.07652	.07542	.07363	.07120	.06818	.06468	.06076	.05654	.05210
.5	.11750	.11695	.11532	.11264	.10900	.10449	.09923	.09336	.08711	.08033
.6	.16473	.16398	.16175	.15810	.15313	.14697	.13977	.13171	.12259	.11380
.7	.21730	.21634	.21349	.20883	.20248	.19459	.18537	.17502	.16380	.15194
.8	.27385	.27269	.26924	.26359	.25587	.24628	.23554	.22240	.20865	.19410
.9	.33302	.33168	.32766	.32108	.31209	.30093	.28773	.27291	.25675	.23956
1.0	.39347	.39196	.38745	.38005	.36992	.35729	.34241	.32561	.30722	.28760
1.1	.45393	.45228	.44736	.43929	.42822	.41439	.39805	.37954	.35922	.33745
1.2	.51325	.51150	.50628	.49770	.48592	.47115	.45360	.43381	.41151	.38835
1.3	.57044	.56863	.56322	.55432	.54207	.52668	.50891	.48756	.46449	.43956
1.4	.62469	.62285	.61737	.60833	.59586	.58016	.56146	.54004	.51623	.49037
1.5	.67535	.67352	.66807	.65907	.64664	.63093	.61216	.59058	.56647	.54014
1.6	.72196	.72018	.71487	.70668	.69390	.6748	.65998	.63861	.61462	.58830
1.7	.76425	.76255	.75746	.74902	.73731	.72242	.70450	.68370	.66023	.63432
1.8	.80210	.80050	.79570	.78774	.77665	.76252	.74542	.72551	.70251	.67781
1.9	.83553	.83040	.82959	.82220	.81187	.79867	.78263	.76384	.74239	.71842
2.0	.86466	.86331	.85925	.85249	.84302	.83086	.81601	.79856	.77650	.75595
2.1	.88975	.88653	.88483	.87879	.87023	.85920	.84568	.82967	.81117	.79023
2.2	.91108	.91000	.90677	.90135	.89373	.88386	.87170	.85723	.84040	.82121
2.3	.92893	.92805	.92523	.92048	.91378	.90507	.89429	.88138	.86627	.84891
2.4	.94387	.94306	.94062	.93652	.93071	.92313	.91369	.90231	.88852	.87341
2.5	.95606	.95538	.95330	.94981	.94484	.93632	.93016	.92027	.90854	.89886
2.6	.96595	.96538	.96364	.96069	.95650	.95096	.94401	.93551	.92536	.91344
2.7	.97388	.97340	.97196	.96952	.96601	.96138	.95551	.94830	.93963	.92936
2.8	.98016	.97977	.97852	.97658	.97370	.96986	.96498	.95893	.95161	.94287
2.9	.98508	.98476	.98381	.98219	.97989	.97670	.97268	.96767	.96156	.95420
3.0	.98889	.98864	.98788	.98658	.98463	.98216	.97801	.97479	.96974	.96362
3.1	.99181	.99161	.99101	.98998	.98849	.98646	.98383	.98152	.97640	.97136
3.2	.99402	.99387	.99340	.99260	.99142	.98982	.98774	.98508	.98176	.97766
3.3	.99568	.99556	.99520	.99458	.99367	.99242	.99073	.98868	.98603	.98273
3.4	.99691	.99682	.99655	.99607	.99537	.99441	.99314	.99149	.98940	.98678
3.5	.99781	.99775	.99754	.99718	.99665	.99532	.99496	.99367	.99203	.98997
3.6	.99847	.99842	.99826	.99800	.99760	.99705	.99631	.99533	.99407	.99246
3.7	.99894	.99890	.99879	.99859	.99829	.99785	.99732	.99659	.99562	.99438
3.8	.99927	.99924	.99916	.99902	.99880	.99850	.99808	.99753	.99646	.99586
3.9	.99950	.99946	.99942	.99932	.99916	.99894	.99864	.99823	.99768	.99697
4.0	.99966	.99965	.99961	.99954	.99942	.99926	.99904	.99874	.99834	.99781
4.1	.99978	.99977	.99974	.99969	.99961	.99949	.99933	.99911	.99882	.99843
4.2	.99985	.99985	.99983	.99979	.99973	.99965	.99954	.99938	.99917	.99888
4.3	.99990	.99990	.99988	.99986	.99982	.99976	.99968	.99957	.99942	.99921
4.4	.99994	.99993	.99992	.99991	.99988	.99984	.99979	.99971	.99960	.99945
4.5	.99996	.99996	.99995	.99994	.99992	.99990	.99986	.99980	.99973	.99962
4.6	.99997	.99997	.99997	.99996	.99995	.99993	.99991	.99987	.99982	.99978
4.7	.99998	.99998	.99998	.99998	.99997	.99996	.99994	.99991	.99988	.99982
4.8	.99999	.99999	.99999	.99998	.99998	.99997	.99996	.99994	.99992	.99988
4.9	.99999	.99999	.99999	.99999	.99999	.99998	.99997	.99996	.99995	.99992

TABLE A-6. (Continued)

ORIGINAL PAGE IS  
OF POOR QUALITY

TABLE A-6. (Continued)

$\lambda$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.1	.00303	.00273	.00243	.00215	.00188	.00162	.00139	.00118	.00099	.00082
.2	.01207	.01088	.00971	.00858	.00750	.00650	.00558	.00474	.00398	.00332
.3	.02699	.02435	.02176	.01926	.01688	.01465	.01259	.01071	.00903	.00753
.4	.04756	.04299	.03850	.03418	.02999	.02609	.02247	.01918	.01621	.01357
.5	.07347	.06657	.05975	.05313	.04660	.04084	.03530	.03022	.02564	.02154
.6	.10434	.0579	.08533	.07611	.06727	.05891	.05112	.04395	.03784	.03160
.7	.13970	.12730	.11498	.10293	.09132	.08030	.06998	.06044	.05174	.04389
.8	.17901	.16369	.14839	.13337	.11883	.10497	.09192	.07979	.06866	.05857
.9	.22170	.20347	.18520	.16718	.14965	.13284	.11692	.10205	.08832	.07578
1.0	.26712	.24614	.22500	.20404	.18315	.16378	.14495	.12724	.11678	.09565
1.1	.31462	.29113	.26734	.24361	.22027	.19762	.17591	.15534	.13618	.11825
1.2	.36353	.33784	.31169	.28545	.25949	.23411	.20962	.18625	.16211	.14354
1.3	.41315	.38568	.35754	.32913	.30082	.27257	.24580	.21384	.19519	.17781
1.4	.46284	.43002	.40433	.37414	.34365	.31382	.28491	.25590	.22858	.20267
1.5	.51196	.48228	.45149	.41997	.38811	.35626	.32486	.29116	.26449	.23110
1.6	.55993	.52987	.49846	.46608	.43310	.39990	.36604	.33428	.30253	.27190
1.7	.60623	.57626	.54473	.51196	.47833	.44420	.40993	.37588	.34239	.30978
1.8	.65041	.62098	.58977	.55710	.52330	.48869	.45365	.41853	.38368	.34994
1.9	.69209	.66359	.63315	.60102	.56750	.53289	.49754	.46179	.42559	.39050
2.0	.73099	.70377	.67447	.64329	.61048	.57632	.54111	.50518	.46886	.43252
2.1	.76689	.74124	.71344	.68353	.65183	.61853	.58389	.54822	.51144	.47508
2.2	.79966	.77580	.74969	.72144	.69118	.65911	.62545	.59047	.55444	.51770
2.3	.82927	.80735	.78316	.75675	.72823	.69772	.66539	.63348	.59623	.55994
2.4	.85574	.83584	.81370	.78932	.75274	.73405	.70337	.67087	.63678	.60134
2.5	.87915	.86130	.84128	.81902	.79454	.76787	.73945	.70831	.67571	.64148
2.6	.89963	.88382	.86591	.84584	.82355	.79904	.77234	.74351	.71268	.68000
2.7	.91737	.90352	.88763	.86379	.84973	.82745	.80296	.77626	.74742	.71657
2.8	.93257	.92057	.90675	.89096	.87310	.85308	.83086	.80640	.77973	.75092
2.9	.94546	.93519	.92324	.90947	.89375	.87556	.85602	.83386	.80946	.78284
3.0	.95624	.94759	.93738	.92550	.91181	.89617	.87887	.85681	.83653	.81220
3.1	.96527	.95799	.94936	.93923	.92744	.91384	.89831	.88069	.86022	.83893
3.2	.97266	.96663	.95942	.95086	.94082	.92912	.91562	.90018	.88267	.86293
3.3	.97868	.97373	.96777	.96063	.95216	.94221	.93611	.91720	.90186	.88445
3.4	.98352	.97951	.97463	.96874	.96168	.95329	.94343	.93192	.91862	.90337
3.5	.98738	.98417	.98022	.97540	.96957	.96259	.95429	.94451	.93310	.91989
3.6	.99042	.98788	.98471	.98082	.97606	.97030	.96340	.95518	.94599	.93416
3.7	.99280	.99080	.98830	.98518	.98134	.97654	.97095	.96411	.95557	.94636
3.8	.99464	.99309	.99112	.98865	.98550	.98179	.97715	.97153	.96476	.95669
3.9	.99605	.99485	.99333	.99140	.98897	.98534	.98221	.97761	.9724	.96534
4.0	.99711	.99620	.99503	.99353	.99163	.98924	.98625	.98256	.97802	.97251
4.1	.99791	.99723	.99634	.99519	.99371	.99184	.98946	.98653	.98266	.97839
4.2	.99850	.99799	.99732	.99645	.99532	.99387	.99203	.98970	.98578	.98317
4.3	.99893	.99856	.99806	.99741	.99655	.99544	.99401	.99219	.98989	.98701
4.4	.99925	.99898	.99861	.99812	.99748	.99664	.99554	.99414	.99234	.99007
4.5	.99948	.99928	.99901	.99865	.99818	.99754	.99672	.99564	.99425	.99248
4.6	.99964	.99950	.99931	.99904	.99869	.99822	.99760	.99679	.99572	.99435
4.7	.99975	.99965	.99952	.99933	.99907	.99873	.99827	.99765	.99685	.99580
4.8	.99983	.99976	.99967	.99953	.99935	.99910	.99876	.99830	.99770	.99690
4.9	.99989	.99984	.99977	.99968	.99955	.99936	.99912	.99878	.99834	.99774

TABLE A-6. (Continued)

ORIGINAL PAGE IS  
OF POOR QUALITY

TABLE A-6. (Continued)

$\lambda$	$\lambda$									
	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.1	.00068	.00055	.00045	.00036	.00028	.00022	.00017	.00013	.00010	.00008
.2	.00273	.00223	.00180	.00144	.00114	.00090	.00071	.00054	.00041	.00031
.3	.00623	.00509	.00413	.00331	.00263	.00207	.00161	.00125	.00095	.00072
.4	.01125	.00924	.00751	.00605	.00483	.00381	.00298	.00231	.00177	.00135
.5	.01793	.01478	.01208	.00977	.00783	.00622	.00489	.00381	.00294	.00225
.6	.02642	.02189	.01797	.01462	.01178	.00940	.00744	.00583	.00452	.00348
.7	.03689	.03074	.02538	.02076	.01683	.01352	.01070	.00849	.00663	.00514
.8	.04952	.04150	.03447	.02838	.02316	.01873	.01501	.01193	.00939	.00732
.9	.06447	.05437	.04546	.03768	.03096	.02522	.02036	.01629	.01292	.01046
1.0	.08189	.06953	.05854	.04886	.04044	.03318	.02699	.02177	.01740	.01379
1.1	.10193	.08714	.07388	.06212	.05179	.04282	.03510	.02853	.02299	.01837
1.2	.12466	.10733	.09166	.07764	.06522	.05434	.04494	.03678	.02918	.02467
1.3	.15014	.13019	.11193	.09558	.08091	.06794	.05658	.04673	.03827	.03108
1.4	.17836	.15576	.13439	.11607	.09901	.08379	.07033	.05856	.04835	.03959
1.5	.20923	.18405	.16067	.13920	.11966	.10205	.08634	.07247	.06033	.04981
1.6	.24263	.21496	.18504	.16500	.14292	.12285	.10476	.08863	.07439	.06192
1.7	.27836	.24836	.22000	.19345	.16804	.14625	.12571	.10770	.09070	.07612
1.8	.31613	.28004	.25342	.22448	.19740	.17229	.14925	.12829	.10941	.09258
1.9	.35564	.32174	.28909	.25794	.22850	.20095	.17541	.15196	.13064	.11143
2.0	.39650	.36113	.32674	.29361	.26200	.23213	.20417	.17824	.15444	.13279
2.1	.43830	.40184	.36604	.33123	.29769	.26568	.23543	.20710	.18083	.15671
2.2	.48059	.44344	.40662	.37046	.33528	.30139	.26903	.23844	.20979	.18321
2.3	.52292	.48551	.44807	.41094	.37447	.33897	.30476	.27209	.24120	.21226
2.4	.56484	.52759	.48935	.45225	.41486	.37812	.34235	.30786	.27451	.24375
2.5	.60590	.56923	.53181	.49397	.45606	.41844	.38146	.34544	.31070	.27751
2.6	.64569	.61000	.57321	.53564	.49763	.45954	.42172	.38453	.34830	.31334
2.7	.68385	.64988	.61371	.57682	.53913	.50098	.46273	.42976	.38736	.35094
2.8	.72006	.68733	.65292	.61109	.58011	.54232	.50406	.46567	.42753	.38999
2.9	.75806	.72322	.69048	.65605	.62037	.58314	.54527	.50690	.46835	.43012
3.0	.78564	.75630	.72608	.69336	.65802	.62301	.58595	.54798	.50953	.47092
3.1	.81467	.78817	.75998	.72870	.69599	.66155	.62562	.58850	.55050	.51197
3.2	.84108	.81691	.79048	.76184	.73110	.69841	.66398	.62809	.59089	.55284
3.3	.86886	.84304	.81895	.79258	.76400	.73331	.70065	.66623	.63028	.59311
3.4	.88605	.86656	.84483	.82081	.79452	.76600	.73535	.70272	.66832	.63237
3.5	.90473	.88751	.86811	.84647	.82253	.79630	.76784	.73720	.70465	.67025
3.6	.92104	.90538	.88884	.86953	.84797	.80411	.79796	.76955	.73900	.70644
3.7	.93512	.92209	.90711	.89007	.87084	.84936	.82557	.79949	.77115	.74064
3.8	.94716	.93539	.92304	.90815	.89119	.87225	.85064	.82693	.80091	.77263
3.9	.95734	.94788	.93679	.92392	.90911	.89223	.87317	.85100	.82820	.80224
4.0	.96587	.95793	.94854	.93753	.92473	.91000	.89319	.87421	.85255	.82938
4.1	.97293	.96635	.95847	.94915	.93821	.92548	.91082	.89409	.87517	.85399
4.2	.97873	.97332	.96679	.95897	.94971	.93883	.92610	.91159	.89453	.87608
4.3	.98344	.97904	.97367	.96719	.95943	.95023	.93941	.92682	.91230	.89571
4.4	.98722	.98368	.97932	.97400	.96757	.95906	.95071	.93996	.92763	.91297
4.5	.99023	.98741	.99390	.97958	.97430	.96791	.96025	.95116	.94086	.92759
4.6	.99260	.99037	.98758	.98411	.97982	.97458	.96823	.96062	.95158	.94093
4.7	.99444	.99271	.99051	.98774	.99430	.98004	.97484	.96853	.96057	.95197
4.8	.99587	.99453	.99281	.99063	.98789	.98447	.98025	.97508	.96861	.96129
4.9	.99696	.99593	.99461	.99291	.99075	.98603	.98463	.98094	.97530	.96908

TABLE A-6. (Continued)

$\lambda$	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
5.0	.99778	.99700	.99599	.99468	.99300	.99085	.98815	.98479	.98062	.97552
5.1	.99839	.99781	.99705	.99605	.99475	.99308	.99095	.98827	.98493	.98079
5.2	.99885	.99842	.99785	.99709	.99610	.99481	.99317	.99164	.98839	.98506
5.3	.99918	.99887	.99844	.99788	.99713	.99614	.99487	.99322	.99113	.98849
5.4	.99943	.99920	.99889	.99847	.99790	.99716	.99619	.99492	.99329	.99121
5.5	.99960	.99944	.99921	.99890	.99849	.99793	.99713	.99623	.99497	.99335
5.6	.99972	.99961	.99944	.99922	.99892	.99850	.99795	.99722	.99626	.99512
5.7	.99981	.99973	.99961	.99945	.99923	.99893	.99852	.99797	.99725	.99630
5.8	.99987	.99982	.99973	.99962	.99946	.99924	.99894	.99854	.99840	.99728
5.9	.99991	.99988	.99982	.99974	.99962	.99947	.99925	.99895	.99855	.9982
6.0	.99994	.99992	.99988	.99982	.99974	.99963	.99947	.99926	.99896	.99857
6.1	.99996	.99994	.99992	.99988	.99982	.99974	.99963	.99948	.99927	.99897
6.2	.99998	.99996	.99995	.99992	.99988	.99983	.99975	.99964	.99948	.99927
6.3	.99998	.99998	.99996	.99995	.99992	.99989	.99983	.99975	.99964	.99949
6.4	.99999	.99998	.99998	.99996	.99995	.99992	.99988	.99983	.99975	.99965
6.5	.99999	.99999	.99998	.99998	.99997	.99995	.99992	.99988	.99983	.99976
6.6	1.00000	.99999	.99999	.99999	.99998	.99997	.99995	.99992	.99989	.99983
6.7	1.00000	1.00000	.99999	.99999	.99999	.99999	.99997	.99995	.99992	.99989
6.8	1.00000	1.00000	1.00000	.99999	.99999	.99999	.99998	.99997	.99995	.99992
6.9	1.00000	1.00000	1.00000	1.00000	.99999	.99999	.99999	.99998	.99997	.99995
7.0	1.00000	1.00000	1.00000	1.00000	1.00000	.99993	.99999	.99999	.99998	.99997
7.1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99999	.99999	.99998
7.2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99999	.99999
7.3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99993	.99999
7.4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999
7.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
7.6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
7.7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
7.8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
7.9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8.9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
9.9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
10.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

ORIGINAL PAGE IS  
OF POOR QUALITY

TABLE A-6. (Continued)

$\lambda$	8	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.1	.00006	.00004	.00003	.00002	.00002	.00001	.00001	.00001	.00001	.00000	.00000
.2	.00023	.00017	.00012	.00009	.00006	.00005	.00003	.00003	.00002	.00002	.00001
.3	.00054	.00040	.00029	.00021	.00015	.00011	.00008	.00008	.00007	.00006	.00003
.4	.00162	.00076	.00056	.00041	.00030	.00021	.00015	.00011	.00007	.00005	.00005
.5	.00170	.00127	.00095	.00070	.00051	.00037	.00026	.00019	.00013	.00009	.00009
.6	.00265	.00200	.00150	.00111	.00081	.00059	.00043	.00030	.00021	.00015	.00015
.7	.00394	.00300	.00226	.00168	.00125	.00091	.00066	.00048	.00034	.00024	.00024
.8	.00566	.00434	.00329	.00248	.00185	.00136	.00100	.00072	.00052	.00037	.00037
.9	.00792	.00611	.00468	.00355	.00267	.00198	.00146	.00117	.00078	.00056	.00056
1.0	.01083	.00843	.00650	.00497	.00377	.00283	.00211	.00155	.00114	.00082	.00082
1.1	.01456	.01142	.00803	.00685	.00524	.00397	.00298	.00222	.00163	.00119	.00119
1.2	.01922	.01521	.01194	.00928	.00716	.00547	.00414	.00311	.00231	.00171	.00171
1.3	.02502	.01997	.01581	.01240	.00965	.00744	.00568	.00430	.00323	.00240	.00240
1.4	.03214	.02588	.02066	.01635	.01282	.00997	.00769	.00587	.00445	.00334	.00334
1.5	.04078	.03311	.02665	.02128	.01684	.01321	.01027	.00792	.00645	.00450	.00450
1.6	.05112	.04186	.03399	.02736	.02184	.01723	.01356	.01555	.00813	.00621	.00621
1.7	.06337	.05232	.04204	.03479	.02801	.02236	.01770	.01388	.01040	.00833	.00833
1.8	.07771	.06470	.05342	.04375	.03553	.02861	.02284	.01868	.01410	.01103	.01103
1.9	.09430	.07916	.06591	.05443	.04458	.03621	.02916	.02328	.01843	.01446	.01446
2.0	.11328	.09588	.08050	.06704	.05537	.04535	.03684	.02967	.02359	.01876	.01876
2.1	.13476	.11438	.09733	.08173	.06807	.05624	.04667	.03743	.03015	.02468	.02468
2.2	.1588C	.13658	.11655	.09868	.08288	.06904	.05704	.04670	.03798	.03060	.03060
2.3	.18591	.16073	.13826	.11801	.09993	.08334	.06934	.05780	.04736	.03849	.03849
2.4	.21454	.18744	.16252	.13983	.11937	.10110	.08494	.07070	.05850	.04795	.04795
2.5	.2461C	.21666	.18932	.16418	.14129	.12060	.10219	.08587	.07157	.05916	.05916
2.6	.27992	.24829	.21862	.19108	.16574	.14265	.12183	.10321	.08675	.07231	.07231
2.7	.31570	.28216	.25032	.22446	.19272	.16719	.14393	.12294	.10417	.08757	.08757
2.8	.35339	.31805	.28424	.25222	.22217	.19925	.16855	.14513	.12398	.10500	.08757
2.9	.39243	.35567	.32016	.28619	.25399	.22370	.19559	.16983	.14626	.12497	.12497
3.0	.43252	.39670	.35780	.32214	.28801	.25566	.22523	.19704	.17104	.14732	.14732
3.1	.47326	.43476	.39682	.35973	.32399	.28972	.25722	.22670	.19832	.17217	.17217
3.2	.51424	.47546	.43686	.39881	.36166	.32574	.29133	.25870	.22809	.19952	.19952
3.3	.55502	.51636	.47751	.43882	.40067	.36341	.32737	.29295	.26009	.22931	.22931
3.4	.59518	.55705	.51835	.47343	.44066	.40242	.36507	.32892	.29428	.26140	.26140
3.5	.63432	.59711	.55896	.52021	.48123	.44240	.40408	.36662	.33038	.29563	.29563
3.6	.67208	.63614	.59893	.56075	.52196	.48293	.44403	.40563	.36810	.33176	.33176
3.7	.71812	.67377	.63785	.60633	.56243	.52361	.48453	.44557	.40711	.36949	.36949
3.8	.74217	.70369	.67537	.63945	.60223	.56402	.52517	.48605	.44703	.40850	.40850
3.9	.77402	.74361	.71116	.67686	.64096	.60370	.56551	.52664	.48768	.44842	.44842
4.0	.80349	.77532	.74495	.71254	.67827	.64239	.60516	.56633	.52803	.48884	.48884
4.1	.83448	.80465	.77654	.74622	.71385	.67960	.64373	.60651	.56827	.52935	.52935
4.2	.85456	.83152	.80575	.77768	.74741	.71500	.68086	.64500	.60779	.56954	.56954
4.3	.87693	.85587	.83249	.80678	.77077	.74059	.71620	.68205	.64621	.60900	.60900
4.4	.89644	.87772	.85673	.83341	.80735	.77979	.74961	.71730	.68317	.64735	.64735
4.5	.91359	.89712	.87847	.85754	.83428	.80867	.78076	.75561	.71938	.68424	.68424
4.6	.92852	.91418	.89777	.87917	.85830	.83509	.80954	.77167	.73597	.71937	.71937
4.7	.94138	.92901	.91473	.89838	.87984	.85902	.83507	.80136	.76254	.72540	.72540
4.8	.95234	.94179	.92949	.91525	.89895	.88047	.85970	.83660	.80115	.78337	.78337
4.9	.96159	.95269	.94218	.92992	.91574	.89950	.88100	.85635	.83730	.81189	.81189

TABLE A-6. (Continued)

$\lambda$	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
5.1	.96932	.96188	.95301	.94255	.93034	.91621	.90001	.88163	.86096	.83796
5.2	.97571	.96955	.96214	.95332	.94290	.93073	.91665	.90052	.88216	.86155
5.3	.98095	.97530	.96977	.96240	.95161	.94323	.93111	.91706	.90096	.88267
5.4	.98519	.98110	.97608	.96998	.96264	.95388	.94351	.93146	.91746	.90146
5.5	.98859	.98530	.98124	.97624	.97017	.96286	.95414	.94385	.93180	.91784
5.6	.99129	.98868	.98542	.98137	.97640	.97036	.96308	.95439	.94413	.93212
5.7	.99341	.99136	.98877	.98552	.98149	.97655	.97093	.96328	.95462	.94439
5.8	.99506	.99346	.99143	.98885	.98562	.98161	.97669	.97070	.96347	.95485
5.9	.99633	.99510	.99352	.99149	.98893	.98571	.98173	.97682	.97085	.96365
6.0	.99803	.99733	.99639	.99513	.99361	.99161	.98907	.98583	.98193	.97706
6.1	.99858	.99805	.99735	.99642	.99521	.99366	.99166	.98913	.98556	.98262
6.2	.99893	.99859	.99807	.99737	.99645	.99525	.99370	.99171	.98913	.98633
6.3	.99928	.99899	.99860	.99808	.99739	.99647	.99523	.99373	.99176	.98925
6.4	.99943	.99923	.99900	.99862	.99810	.99741	.99650	.99531	.99377	.99179
6.5	.99965	.99950	.99929	.99901	.99863	.99811	.99742	.99652	.99533	.99379
6.6	.99976	.99965	.99950	.99930	.99902	.99864	.99812	.99744	.99653	.99534
6.7	.99983	.99976	.99966	.99951	.99930	.99902	.99864	.99812	.99745	.99654
6.8	.99989	.99984	.99976	.99966	.99951	.99931	.99903	.99865	.99814	.99745
6.9	.99993	.99985	.99984	.99976	.99966	.99951	.99931	.99903	.99865	.99813
7.0	.99995	.99993	.99989	.99984	.99987	.99966	.99952	.99931	.99903	.99864
7.1	.99997	.99995	.99993	.99989	.99984	.99977	.99966	.99952	.99931	.99902
7.2	.99993	.99997	.99995	.99993	.99989	.99984	.99977	.99966	.99951	.99929
7.3	.99999	.99998	.99997	.99995	.99993	.99989	.99984	.99976	.99965	.99949
7.4	.99999	.99999	.99993	.99997	.99995	.99993	.99983	.99984	.99975	.99963
7.5	.99999	.99999	.99993	.99998	.99987	.99995	.99993	.99989	.99983	.99973
7.6	1.00000	.99999	.99999	.99999	.99998	.99997	.99995	.99992	.99988	.99980
7.7	1.00000	1.00000	.99993	.99999	.99999	.99998	.99996	.99994	.99991	.99985
7.8	1.00000	1.00000	1.00000	.99999	.99999	.99999	.99998	.99996	.99993	.99989
7.9	1.00000	1.00000	1.00000	1.00000	.99999	.99999	.99998	.99997	.99995	.99991
8.0	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99999	.99998	.99996	.99992
8.1	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99999	.99998	.99997	.99993
8.2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99997	.99994
8.3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99997	.99995
8.4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99997	.99995
8.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99998	.99995
8.6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99998	.99995
8.7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
8.8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
8.9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
9.9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995
10.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99995

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TABLE A-6. (Continued)

	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9
5.0	.81259	.78490	.75495	.72288	.68887	.65319	.61599	.57771	.53868	.49924
5.1	.83853	.81327	.78561	.75570	.72365	.68566	.65394	.61679	.57851	.51594
5.2	.86210	.83919	.81331	.83975	.81451	.78632	.72438	.69040	.65869	.61752
5.3	.88316	.86263	.86313	.88405	.88445	.81508	.75706	.72506	.69108	.65536
5.4	.90182	.90221	.88361	.90258	.90292	.88400	.78751	.75767	.72615	.69166
5.5	.91819	.91853	.91888	.93270	.91912	.90321	.84122	.79804	.75810	.72645
5.6	.93292	.94465	.94488	.93295	.93318	.91938	.88438	.86454	.82179	.78872
5.7	.95505	.95524	.95509	.95526	.94525	.93333	.91947	.90348	.88522	.84176
5.8	.96382	.96397	.95540	.95552	.95535	.95556	.93339	.91944	.86473	.83390
5.9	.97113	.97125	.96403	.97133	.96416	.95546	.94533	.93326	.90279	.86358
6.0	.97717	.97725	.97730	.97729	.97136	.96429	.95398	.94510	.90273	.90165
6.1	.98210	.98216	.98218	.98218	.98213	.97717	.97105	.97054	.96167	.91627
6.2	.98609	.98612	.98612	.98612	.98604	.98197	.97687	.97627	.96126	.95098
6.3	.98928	.99182	.98930	.99181	.99176	.98916	.98892	.98852	.96794	.95885
6.4	.99182	.99378	.99531	.99372	.99353	.99356	.99135	.99269	.97515	.96530
6.5	.99380	.99534	.99649	.99523	.99505	.99613	.99471	.99412	.97778	.97054
6.6	.99653	.99743	.99738	.99633	.99727	.99706	.99583	.99521	.99128	.97475
6.7	.99811	.99811	.99805	.99793	.99771	.99688	.99688	.99603	.98336	.97867
7.0	.99861	.99898	.99855	.99842	.99819	.99731	.99664	.99553	.98623	.98067
7.1	.99926	.99932	.99918	.99904	.99859	.99812	.99708	.99694	.98463	.98268
7.2	.99945	.99953	.99938	.99923	.99838	.99836	.99781	.99747	.98781	.98021
7.3	.99955	.99951	.99951	.99937	.99911	.99854	.99764	.99712	.99020	.98905
7.4	.99963	.99969	.99961	.99946	.99920	.99806	.99781	.99732	.99152	.98957
7.5	.99976	.99972	.99968	.99957	.99930	.99874	.99792	.99751	.99252	.98928
7.6	.99981	.99986	.99980	.99976	.99953	.99926	.99880	.99800	.99492	.98622
7.7	.99986	.99986	.99980	.99975	.99960	.99933	.99887	.99809	.99326	.98685
7.8	.99986	.99986	.99980	.99976	.99957	.99930	.99884	.99826	.99420	.98731
7.9	.99986	.99986	.99980	.99975	.99962	.99935	.99887	.99826	.99159	.98764
8.0	.99988	.99988	.99980	.99975	.99962	.99936	.99887	.99826	.99180	.98803
8.1	.99989	.99989	.99980	.99975	.99964	.99936	.99887	.99826	.99180	.98803
8.2	.99989	.99989	.99980	.99975	.99964	.99936	.99887	.99826	.99180	.98803
8.3	.99990	.99990	.99981	.99981	.99963	.99937	.99880	.99826	.99211	.98814
8.4	.99990	.99990	.99981	.99981	.99965	.99938	.99883	.99826	.99211	.98821
8.5	.99990	.99990	.99981	.99982	.99966	.99938	.99884	.99826	.99223	.98826
8.6	.99990	.99990	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
8.7	.99990	.99990	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
8.8	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
8.9	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.0	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.1	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.2	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.3	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.4	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.5	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.6	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.7	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.8	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
9.9	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836
10.0	.99991	.99991	.99982	.99982	.99966	.99938	.99884	.99826	.99231	.98836

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TABLE A-6. (Continued)

$\lambda$	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9
.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.1	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.2	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.3	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.4	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.5	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.6	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.7	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.8	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.9	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.0	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.1	.00002	.00001	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000
1.2	.00013	.00002	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000
1.3	.00005	.00003	.00002	.00001	.00001	.00001	.00000	.00000	.00000	.00000
1.4	.00008	.00005	.00003	.00002	.00001	.00001	.00001	.00000	.00000	.00000
1.5	.00012	.00008	.00005	.00004	.00002	.00002	.00001	.00001	.00000	.00000
1.6	.00016	.00012	.00008	.00006	.00004	.00002	.00002	.00001	.00001	.00000
1.7	.00026	.00018	.00012	.00008	.00006	.00004	.00002	.00002	.00001	.00001
1.8	.00039	.00027	.00019	.00013	.00009	.00006	.00004	.00003	.00002	.00002
1.9	.00056	.00039	.00027	.00019	.00013	.00009	.00006	.00004	.00003	.00002
2.0	.00080	.00057	.00046	.00028	.00019	.00013	.00009	.00006	.00004	.00003
2.1	.00114	.00081	.00059	.00041	.00028	.00020	.00014	.00009	.00006	.00004
2.2	.00159	.00115	.00083	.00059	.00042	.00029	.00021	.00014	.00009	.00006
2.3	.00221	.00162	.00117	.00080	.00060	.00042	.00029	.00020	.00014	.00009
2.4	.00304	.00228	.00164	.00119	.00085	.00061	.00043	.00030	.00021	.00014
2.5	.00414	.00308	.00227	.00166	.00120	.00086	.00062	.00043	.00030	.00021
2.6	.00557	.00419	.00312	.00230	.00168	.00122	.00088	.00062	.00044	.00031
2.7	.00743	.00563	.00423	.00315	.00233	.00170	.00123	.00093	.00063	.00044
2.8	.00981	.00751	.00569	.00428	.00319	.00235	.00171	.00125	.00093	.00064
2.9	.01283	.00991	.00753	.00575	.00432	.00322	.00238	.00170	.00126	.00091
3.0	.01662	.01295	.01000	.00765	.00580	.00436	.00325	.00240	.00176	.00127
3.1	.02132	.01676	.01306	.01008	.00772	.00586	.00440	.00328	.00242	.00177
3.2	.02709	.02148	.01667	.01316	.01017	.00778	.00592	.00444	.00331	.00244
3.3	.03411	.02729	.02164	.01702	.01326	.01024	.00786	.00595	.00449	.00334
3.4	.04255	.03434	.02748	.02180	.01714	.01336	.01037	.00793	.00592	.00451
3.5	.05260	.04282	.03456	.02766	.02190	.01726	.01345	.01039	.00795	.00600
3.6	.06445	.05231	.04309	.03478	.02783	.02200	.01737	.01350	.01046	.00801
3.7	.07825	.06480	.05321	.04333	.03498	.02900	.02222	.01767	.01362	.01053
3.8	.09419	.07865	.06514	.05365	.04357	.03281	.02234	.01758	.01371	.01064
3.9	.11238	.09663	.07903	.06546	.05377	.04379	.03356	.02831	.02287	.01767
4.0	.13293	.11287	.09567	.07940	.06577	.05473	.04466	.03554	.02845	.02298
4.1	.15596	.13349	.11330	.09546	.07935	.06606	.05427	.04421	.03871	.02889
4.2	.18144	.15655	.13463	.11380	.09585	.08000	.06635	.05451	.04441	.03587
4.3	.20937	.18207	.15711	.13450	.11422	.09622	.06639	.05681	.05073	.04459
4.4	.23966	.21004	.18267	.15764	.13497	.11463	.09653	.06869	.06646	.05493
4.5	.27219	.24037	.21067	.18324	.15814	.13540	.11501	.09689	.06975	.06707
4.6	.30676	.27292	.24103	.21127	.18377	.15860	.13587	.11534	.09717	.08137
4.7	.34313	.30751	.27360	.24164	.21182	.18425	.15901	.13630	.11562	.09738
4.8	.38059	.34388	.30827	.27423	.24220	.21229	.18467	.15934	.13639	.11584
4.9	.42000	.38173	.34056	.30881	.27476	.24265	.21266	.18493	.15953	.13656

TABLE A-6. (Concluded)

$\lambda$	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9
5.0	.45980	.42072	.38238	.34514	.30931	.27517	.24296	.21287	.18543	.15953
5.1	.49997	.46047	.42133	.38291	.34597	.30962	.27537	.24363	.21201	.18484
5.2	.54011	.50058	.46100	.42175	.38322	.34579	.30965	.27529	.24279	.21236
5.3	.57980	.50062	.50099	.46129	.42190	.38320	.34553	.30926	.27462	.24192
5.4	.61864	.50019	.50089	.50110	.46122	.42161	.38266	.34470	.30818	.27330
5.5	.65625	.61887	.50026	.50076	.50029	.46057	.42065	.38137	.30310	.30620
5.6	.69229	.65629	.61870	.57983	.54002	.49963	.45957	.413872	.37900	.34029
5.7	.72696	.69203	.65582	.61790	.57863	.53036	.49766	.45635	.41549	.37516
5.8	.75851	.72520	.69127	.65458	.61615	.57630	.53539	.49380	.45159	.41037
5.9	.78824	.75771	.72075	.68952	.65219	.61304	.57233	.53062	.48815	.44596
6.0	.81559	.78709	.75603	.72292	.68640	.64810	.60805	.56635	.52350	.47998
6.1	.84031	.81399	.79490	.75304	.71849	.68139	.66199	.60059	.55760	.51398
6.2	.86255	.83834	.81124	.78120	.75022	.71238	.67388	.63297	.59004	.54554
6.3	.88223	.86013	.83500	.80679	.77543	.74095	.70396	.66120	.62050	.57580
6.4	.89961	.87940	.85619	.82976	.80002	.76653	.73054	.69103	.65970	.61398
6.5	.91463	.89624	.87482	.85012	.82137	.79027	.75501	.71632	.67946	.62983
6.6	.92750	.91070	.89103	.86795	.84131	.80195	.77682	.73898	.69766	.65323
6.7	.93841	.92319	.90496	.88337	.85813	.82904	.79593	.75900	.71425	.67949
6.8	.94754	.93365	.91677	.89652	.87257	.84866	.81262	.77645	.73628	.69242
6.9	.95559	.94235	.92666	.90760	.88880	.85795	.82605	.79145	.75184	.70381
7.0	.96125	.94951	.93484	.91681	.89562	.86911	.83881	.80415	.76517	.72187
7.1	.96624	.95531	.94152	.92437	.90345	.87416	.84804	.81476	.77616	.73327
7.2	.97200	.95997	.94690	.93049	.91030	.88192	.85707	.82350	.78532	.74273
7.3	.97332	.96365	.95117	.93538	.91580	.89200	.85366	.82059	.79279	.75065
7.4	.97579	.96652	.95453	.93924	.92016	.89680	.86394	.83626	.79377	.75666
7.5	.97760	.96873	.95712	.94223	.92335	.90063	.87305	.84073	.80351	.76159
7.6	.97900	.97042	.95911	.94453	.92617	.90355	.87631	.84421	.80720	.76544
7.7	.98005	.97163	.96060	.94627	.92815	.90579	.87277	.84687	.81063	.76600
7.8	.98082	.97262	.96171	.94756	.92564	.90795	.88062	.84888	.81218	.77068
7.9	.98139	.97330	.96253	.94852	.93074	.90869	.88199	.85030	.81378	.77233
8.0	.98179	.97379	.96312	.94971	.93153	.90960	.88300	.85207	.81955	.77356
8.1	.98208	.97319	.96354	.94971	.93211	.91025	.88372	.85227	.81580	.77446
8.2	.98228	.97439	.96383	.95016	.93251	.91271	.89424	.85283	.81641	.77510
8.3	.98242	.97456	.96404	.95030	.93280	.91103	.88460	.85233	.81604	.77556
8.4	.98251	.97468	.96419	.95047	.93295	.91126	.88485	.85350	.81730	.77587
8.5	.98254	.97475	.96427	.95058	.93312	.91141	.88502	.85363	.81734	.77609
8.6	.98262	.97481	.96439	.95066	.93321	.91151	.88513	.85382	.81748	.77623
8.7	.98264	.97488	.96431	.95071	.93327	.91158	.88521	.85390	.81757	.77633
8.8	.98266	.97486	.96441	.95079	.93331	.91162	.88526	.85396	.81763	.77639
8.9	.98267	.97488	.96442	.95076	.93333	.91165	.88529	.85409	.81767	.77643
9.0	.98268	.97489	.96443	.95077	.93335	.91167	.88531	.8542	.81769	.77646
9.1	.98269	.97489	.96444	.95078	.93336	.91168	.88532	.8543	.81771	.77648
9.2	.98269	.97489	.96445	.95078	.93337	.91169	.88533	.85409	.81772	.77649
9.3	.98269	.97490	.96445	.95079	.93337	.91169	.88530	.85409	.81772	.77649
9.4	.98269	.97490	.96445	.95079	.93337	.91169	.88530	.85405	.81772	.77650
9.5	.98269	.97490	.96445	.95079	.93337	.91169	.88530	.85405	.81773	.77650
9.6	.98269	.97490	.96445	.95079	.93337	.91169	.88530	.85405	.81773	.77650
9.7	.98269	.97490	.96445	.95079	.93337	.91169	.88530	.85405	.81773	.77650
9.8	.98269	.97490	.96445	.95079	.93337	.91169	.88530	.85405	.81773	.77650
9.9	.98269	.97490	.96445	.95079	.93337	.91169	.88530	.85405	.81773	.77650
10.0	.98269	.97490	.96445	.95079	.93337	.91169	.88530	.85405	.81773	.77650

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## APPROVAL

### DISTRIBUTION AND MOMENTS OF RADIAL ERROR

By Robert G. White

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This document has also been reviewed and approved for technical accuracy.



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